5th

Total No. of Printed Pages—5

1 SEM TDC MTMH (CBCS) C 2

2024

(November)

MATHEMATICS

(Core)

Paper : C–2

(Algebra)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Find $|4+3i|$. 1

(b) Express $\dfrac{3-2i}{-1+i}$ in the form $x+iy$. 2

(c) If

$$\sin\alpha+\sin\beta+\sin\gamma=\cos\alpha+\cos\beta+\cos\gamma=0$$

prove that

$$\cos 3\alpha+\cos 3\beta+\cos 3\gamma=3\cos(\alpha+\beta+\gamma)$$

and

$$\sin 3\alpha+\sin 3\beta+\sin 3\gamma=3\sin(\alpha+\beta+\gamma)$$ 3

(d) Find the cube root of $8i$. 4

2\. (a) State the well-ordering property of positive integers. 1

(b) Let $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=\sin x$ and $g:\mathbb{R}\to\mathbb{R}$ given by $g(x)=x^2$. Then find $(f\circ g)(x)$ and $(g\circ f)(x)$. 1

(c) If $f:A\to B$ be one-one onto, then prove that inverse mapping of f is unique. 2

(d) Consider $\mathbb{N}\times\mathbb{N}$ be the set of orderrd pairs of natural numbers. Let $\mathcal{R}$ be the relation in $\mathbb{N}\times\mathbb{N}$ defined by $(a, b)\mathcal{R}(c, d)$ iff $a+d=b+c$. Then show that $\mathcal{R}$ is an equivalence relation. 3

(e) Show that $f:X\to Y$ be invertible iff f is a bijection. Also show that $g:\mathbb{R}\to\mathbb{R}$ defined by $g(x)=3x+5$ is a bijection and find its inverse. 3+2+1=6

(f) State and prove the division algorithm. 1+4=5

Or

Prove by mathematical induction that n^5-n is divisible by 30. 5

(g) Find the g.c.d. of 1166 and 256 and express it in the form

$$1166x+256y=(1166,\ 256)$$ 4

(h) If $(a, c)=(b, c)=1$, then prove that $(ab, c)=1$. 3

3\. (a) Express $v=(1, 3, 2)$ as a linear combination of $u_1=(1, 2, 1)$, $u_2=(2, 6, 5)$ and $u_3=(1, 7, 8)$. 2

(b) Solve : 2

$$2x-6y+7z=1$$
$$4y+3z=8$$
$$2z=4$$

(c) What do you mean by linear dependence of vectors? Show that the vectors $(1, 2, -3, 4)$, $(3, -1, 2, 1)$ and $(1, -5, 8, -7)$ are linearly dependent. 1+3=4

(d) Find for what values of λ and μ, the system of equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

has *(i)* no solution, *(ii)* a unique solution and *(iii)* an infinite number of solutions. 2+2+2=6

(e) Find the row-reduced echelon form of

$$A=\begin{bmatrix}2 & -1 & 3\\ 3 & 2 & 1\\ 1 & -4 & 5\end{bmatrix}$$

and hence find the rank of it. 6

Or

Reduce $A = \begin{bmatrix} \frac{1}{3} & \frac{3}{2} & 3 & -8 \\ \frac{2}{3} & 3 & 2 & 1 \\ 1 & \frac{9}{2} & 5 & -7 \\ \frac{4}{3} & 6 & 8 & -15 \end{bmatrix}$ to its normal form.

4. (a) Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$F(x, y) = (x - y, x - 2y)$$

Determine whether F is singular or non-singular. 2

(b) Find the matrix representation of the linear transformation T on $\mathbb{R}^3$ given by $T(x, y, z) = (x, y, 0)$. 3

(c) Find the eigenvalues of

$$A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{bmatrix}$$ 3

(d) Let

$$P = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

(i) Find the eigenvectors corresponding to the eigenvalues.

(ii) Find the characteristic polynomial of P. 2+2=4

(e) Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$$ 5

Or

Find a linear map $T : \mathbb{R}^3 \to \mathbb{R}^4$ whose image is spanned by $(1, 2, 0, -4)$ and $(2, 0, -1, -3)$.

(f) Show that a matrix A and its transpose A^T have the same characteristic polynomial. 3

Or

Let $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$. Find $f(A)$, where $f(t) = t^2 - 3t + 7$.

(g) Let W be a subspace of $\mathbb{R}^4$ spanned by the vectors $u_1 = (1, -2, 5, -3)$, $u_2 = (2, 3, 1, -4)$, $u_3 = (3, 8, -3, -5)$. Find a basis and dimension of W. 5

Or

Determine whether $(1, 1, 1, 1)$, $(1, 2, 3, 2)$, $(2, 5, 6, 4)$, $(2, 6, 8, 5)$ form a basis of $\mathbb{R}^4$. If not, find the dimension of subspace they span.

★★★

Total No. of Printed Pages—5

1 SEM TDC MTMH (CBCS) C 2

2024

(November)

MATHEMATICS

(Core)

Paper : C–2

(Algebra)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Find $|4+3i|$. 1

(b) Express $\dfrac{3-2i}{-1+i}$ in the form $x+iy$. 2

(c) If

$$\sin\alpha+\sin\beta+\sin\gamma=\cos\alpha+\cos\beta+\cos\gamma=0$$

prove that

$$\cos 3\alpha+\cos 3\beta+\cos 3\gamma=3\cos(\alpha+\beta+\gamma)$$

and

$$\sin 3\alpha+\sin 3\beta+\sin 3\gamma=3\sin(\alpha+\beta+\gamma)$$ 3

(d) Find the cube root of $8i$. 4

2. *(a)* State the well-ordering property of positive integers. 1

(b) Let $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=\sin x$ and $g:\mathbb{R}\to\mathbb{R}$ given by $g(x)=x^2$. Then find $(f\circ g)(x)$ and $(g\circ f)(x)$. 1

(c) If $f:A\to B$ be one-one onto, then prove that inverse mapping of f is unique. 2

(d) Consider $\mathbb{N}\times\mathbb{N}$ be the set of orderrd pairs of natural numbers. Let $\mathcal{R}$ be the relation in $\mathbb{N}\times\mathbb{N}$ defined by $(a, b)\mathcal{R}(c, d)$ iff $a+d=b+c$. Then show that $\mathcal{R}$ is an equivalence relation. 3

(e) Show that $f:X\to Y$ be invertible iff f is a bijection. Also show that $g:\mathbb{R}\to\mathbb{R}$ defined by $g(x)=3x+5$ is a bijection and find its inverse. 3+2+1=6

(f) State and prove the division algorithm. 1+4=5

Or

Prove by mathematical induction that n^5-n is divisible by 30. 5

(g) Find the g.c.d. of 1166 and 256 and express it in the form

$$1166x+256y=(1166, 256)$$

4

(h) If $(a, c)=(b, c)=1$, then prove that $(ab, c)=1$. 3

3. *(a)* Express $v=(1, 3, 2)$ as a linear combination of $u_1=(1, 2, 1)$, $u_2=(2, 6, 5)$ and $u_3=(1, 7, 8)$. 2

(b) Solve : 2

$$\begin{aligned} 2x-6y+7z&=1\\ 4y+3z&=8\\ 2z&=4 \end{aligned}$$

(c) What do you mean by linear dependence of vectors? Show that the vectors $(1, 2, -3, 4)$, $(3, -1, 2, 1)$ and $(1, -5, 8, -7)$ are linearly dependent. 1+3=4

(d) Find for what values of λ and μ, the system of equations

$$\begin{aligned} x+y+z&=6\\ x+2y+3z&=10\\ x+2y+\lambda z&=\mu \end{aligned}$$

has *(i)* no solution, *(ii)* a unique solution and *(iii)* an infinite number of solutions. 2+2+2=6

(e) Find the row-reduced echelon form of

$$A=\begin{bmatrix} 2 & -1 & 3\\ 3 & 2 & 1\\ 1 & -4 & 5 \end{bmatrix}$$

and hence find the rank of it. 6

Or

Reduce $A = \begin{bmatrix} \frac{1}{3} & \frac{3}{2} & 3 & -8 \\ \frac{2}{3} & 3 & 2 & 1 \\ 1 & \frac{9}{2} & 5 & -7 \\ \frac{4}{3} & 6 & 8 & -15 \end{bmatrix}$ to its normal form.

4. *(a)* Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$F(x, y) = (x - y, x - 2y)$$

Determine whether F is singular or non-singular. 2

(b) Find the matrix representation of the linear transformation T on $\mathbb{R}^3$ given by $T(x, y, z) = (x, y, 0)$. 3

(c) Find the eigenvalues of

$$A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{bmatrix}$$ 3

(d) Let

$$P = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

(i) Find the eigenvectors corresponding to the eigenvalues.

(ii) Find the characteristic polynomial of *P*. 2+2=4

(e) Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$$ 5

Or

Find a linear map $T : \mathbb{R}^3 \to \mathbb{R}^4$ whose image is spanned by (1, 2, 0, –4) and (2, 0, –1, –3).

(f) Show that a matrix A and its transpose A^T have the same characteristic polynomial. 3

Or

Let $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$. Find $f(A)$, where $f(t) = t^2 - 3t + 7$.

(g) Let W be a subspace of $\mathbb{R}^4$ spanned by the vectors $u_1 = (1, -2, 5, -3)$, $u_2 = (2, 3, 1, -4)$, $u_3 = (3, 8, -3, -5)$. Find a basis and dimension of W. 5

Or

Determine whether (1, 1, 1, 1), (1, 2, 3, 2), (2, 5, 6, 4), (2, 6, 8, 5) form a basis of $\mathbb{R}^4$. If not, find the dimension of subspace they span.

★★★

Total No. of Printed Pages—4

1 SEM TDC MTMH (CBCS) C 1

2024

(November)

MATHEMATICS

(Core)

Paper : C–1

(Calculus)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write the domain of definition of the function $\cosh^{-1} x$. 1

(b) Write the necessary condition for the function $f(x)$ to have an extreme value at $x = c$. 1

(c) Find y_n if $y = e^{ax} \cos bx$. 3

Or

If $y = a\cos(\log x) + b\sin(\log x)$, show that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

(d) Find the asymptote of the curve

$$y = \frac{x^2}{x^2+1}$$

parallel to x-axis. 3

(e) Evaluate any *one* of the following : 3

(i) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

(ii) $\lim_{x \to 2} \frac{x^7 - 128}{x^3 - 8}$

(f) Find the range of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave up or concave down. Also determine the points of inflection. 3+1=4

(g) Trace the curve $y = x^3 - 12x - 16$. 5

Or

A manufacturer estimates that when x units of a particular commodity are produced each month, the total cost (in rupees) will be $C(x) = \frac{1}{8}x^2 + 4x + 200$ and all units can be sold at a price of $p(x) = 49 - x$ rupees per unit. Determine the price that corresponds to the maximum profit.

2. Answer any *three* of the following : 5×3=15

(a) Obtain the reduction formula for

$$\int_0^{\pi/2} \sin^n x \, dx$$

(Continued)

(b) Find the volume and area of curved surface of a paraboloid of revolution formed by revolving the parabola $y^2 = 4ax$ about the x-axis and bounded by the section $x = x_1$.

(c) Find the volume and the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.

(d) Show that

$$I_n = \int_0^\infty \frac{dx}{(1+x^2)^n} = \frac{2n-3}{2n-2} I_{n-1}$$

(e) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$ and the x-axis is revolved about the y-axis.

3. (a) Suppose that the axes of any xy-coordinate system are rotated through an angle of $\theta = 45°$ to obtain an $x'y'$-coordinate system. Find the equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$-coordinate. 5

(b) Answer any *two* of the following : 4×2=8

(i) Find the arc length of the curve $y = x^{3/2}$ from (1, 1) to $(2, 2\sqrt{2})$.

(ii) Sketch the graph of the ellipse $x^2 + 2y^2 = 4$ showing the foci.

(Turn Over)

(iii) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis.

(c) Find the new coordinates of the point (2, 4) if the coordinate axes are rotated through an angle $\theta = 30°$. 2

4. *(a)* If $\vec{r} = \vec{a}\cos\omega t + \vec{b}\sin\omega t$, then show that

$$\frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r}$$

2

(b) Prove that

$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$$

2

(c) Answer any *two* of the following : 3×2=6

(i) Find the unit tangent vector at any point on the curve $x = a\cos t$, $y = a\sin t$ and $z = bt$.

(ii) Find $\lim_{t \to 1} [\vec{F}(t) \times \vec{G}(t)]$, where

$$\vec{F}(t) = t\hat{i} + (1 - t)\hat{j} + t^2\hat{k}$$

and $\vec{G}(t) = e^t\hat{i} - (3 + e^t)\hat{k}$

(iii) Find

$$\int_0^{\pi} [t\hat{i} + 3\hat{j} - (\sin t)\hat{k}]\, dt$$

★★★

1 SEM TDC MTMH (CBCS) C 1

2024

(November)

MATHEMATICS

(Core)

Paper : C–1

(Calculus)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write the domain of definition of the function $\cosh^{-1} x$. 1

(b) Write the necessary condition for the function $f(x)$ to have an extreme value at $x = c$. 1

(c) Find y_n if $y = e^{ax} \cos bx$. 3

Or

If $y = a\cos(\log x) + b\sin(\log x)$, show that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

(d) Find the asymptote of the curve

$$y = \frac{x^2}{x^2+1}$$

parallel to x-axis. 3

(e) Evaluate any *one* of the following : 3

(i) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

(ii) $\lim_{x \to 2} \frac{x^7 - 128}{x^3 - 8}$

(f) Find the range of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave up or concave down. Also determine the points of inflection. 3+1=4

(g) Trace the curve $y = x^3 - 12x - 16$. 5

Or

A manufacturer estimates that when x units of a particular commodity are produced each month, the total cost (in rupees) will be $C(x) = \frac{1}{8}x^2 + 4x + 200$ and all units can be sold at a price of $p(x) = 49 - x$ rupees per unit. Determine the price that corresponds to the maximum profit.

2. Answer any *three* of the following : 5×3=15

(a) Obtain the reduction formula for

$$\int_0^{\pi/2} \sin^n x \, dx$$

(b) Find the volume and area of curved surface of a paraboloid of revolution formed by revolving the parabola $y^2 = 4ax$ about the x-axis and bounded by the section $x = x_1$.

(c) Find the volume and the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.

(d) Show that

$$I_n = \int_0^\infty \frac{dx}{(1+x^2)^n} = \frac{2n-3}{2n-2} I_{n-1}$$

(e) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$ and the x-axis is revolved about the y-axis.

3. (a) Suppose that the axes of any xy-coordinate system are rotated through an angle of $\theta = 45°$ to obtain an $x'y'$-coordinate system. Find the equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$-coordinate. 5

(b) Answer any *two* of the following : 4×2=8

(i) Find the arc length of the curve $y = x^{3/2}$ from (1, 1) to $(2, 2\sqrt{2})$.

(ii) Sketch the graph of the ellipse $x^2 + 2y^2 = 4$ showing the foci.

(iii) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis.

(c) Find the new coordinates of the point (2, 4) if the coordinate axes are rotated through an angle $\theta = 30°$. 2

4. *(a)* If $\vec{r} = \vec{a}\cos\omega t + \vec{b}\sin\omega t$, then show that

$$\frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r}$$

2

(b) Prove that

$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$$

2

(c) Answer any *two* of the following : 3×2=6

(i) Find the unit tangent vector at any point on the curve $x = a\cos t$, $y = a\sin t$ and $z = bt$.

(ii) Find $\lim_{t \to 1} [\vec{F}(t) \times \vec{G}(t)]$, where

$$\vec{F}(t) = t\hat{i} + (1-t)\hat{j} + t^2\hat{k}$$

and $\vec{G}(t) = e^t\hat{i} - (3 + e^t)\hat{k}$

(iii) Find

$$\int_0^{\pi} [t\hat{i} + 3\hat{j} - (\sin t)\hat{k}]\, dt$$

★★★

1 SEM TDC MTMH (CBCS) C 1

2024

(November)

MATHEMATICS

(Core)

Paper : C–1

(Calculus)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write the domain of definition of the function $\cosh^{-1} x$. 1

(b) Write the necessary condition for the function $f(x)$ to have an extreme value at $x = c$. 1

(c) Find y_n if $y = e^{ax} \cos bx$. 3

Or

If $y = a\cos(\log x) + b\sin(\log x)$, show that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

(d) Find the asymptote of the curve

$$y = \frac{x^2}{x^2+1}$$

parallel to x-axis. 3

(e) Evaluate any *one* of the following : 3

(i) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

(ii) $\lim_{x \to 2} \frac{x^7 - 128}{x^3 - 8}$

(f) Find the range of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave up or concave down. Also determine the points of inflection. 3+1=4

(g) Trace the curve $y = x^3 - 12x - 16$. 5

Or

A manufacturer estimates that when x units of a particular commodity are produced each month, the total cost (in rupees) will be $C(x) = \frac{1}{8}x^2 + 4x + 200$ and all units can be sold at a price of $p(x) = 49 - x$ rupees per unit. Determine the price that corresponds to the maximum profit.

2. Answer any *three* of the following : 5×3=15

(a) Obtain the reduction formula for

$$\int_0^{\pi/2} \sin^n x \, dx$$

(b) Find the volume and area of curved surface of a paraboloid of revolution formed by revolving the parabola $y^2 = 4ax$ about the x-axis and bounded by the section $x = x_1$.

(c) Find the volume and the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.

(d) Show that

$$I_n = \int_0^\infty \frac{dx}{(1+x^2)^n} = \frac{2n-3}{2n-2} I_{n-1}$$

(e) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$ and the x-axis is revolved about the y-axis.

3. *(a)* Suppose that the axes of any xy-coordinate system are rotated through an angle of $\theta = 45°$ to obtain an $x'y'$-coordinate system. Find the equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$-coordinate. 5

(b) Answer any *two* of the following : 4×2=8

(i) Find the arc length of the curve $y = x^{3/2}$ from $(1, 1)$ to $(2, 2\sqrt{2})$.

(ii) Sketch the graph of the ellipse $x^2 + 2y^2 = 4$ showing the foci.

(iii) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis.

(c) Find the new coordinates of the point (2, 4) if the coordinate axes are rotated through an angle $\theta = 30°$. 2

4. *(a)* If $\vec{r} = \vec{a}\cos\omega t + \vec{b}\sin\omega t$, then show that

$$\frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r}$$ 2

(b) Prove that

$$\hat{i}\times(\vec{a}\times\hat{i}) + \hat{j}\times(\vec{a}\times\hat{j}) + \hat{k}\times(\vec{a}\times\hat{k}) = 2\vec{a}$$ 2

(c) Answer any *two* of the following : 3×2=6

(i) Find the unit tangent vector at any point on the curve $x = a\cos t$, $y = a\sin t$ and $z = bt$.

(ii) Find $\lim_{t\to 1}[\vec{F}(t)\times\vec{G}(t)]$, where

$$\vec{F}(t) = t\hat{i} + (1-t)\hat{j} + t^2\hat{k}$$

and $\vec{G}(t) = e^t\hat{i} - (3+e^t)\hat{k}$

(iii) Find

$$\int_0^{\pi}[t\hat{i} + 3\hat{j} - (\sin t)\hat{k}]\,dt$$

★ ★ ★