Total number of printed pages–7

2 SEM PG (CBCS) MTH C 4

2025

(June)

MATHEMATICS

Paper : MTH C4

(Complex Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

UNIT–I

Answer Question No. **1** *and* ***any two*** *from the rest :* 4×3=12

1. Examine whether $f(z) = \frac{i}{z^2}$ is analytic, whenever $z \neq 0$. Find components and check whether the components are harmonic or not.

2. Find the principal argument of the following :

(i) $\frac{i}{-2-2i}$

(ii) $(\sqrt{3}-i)^6$

3. Find the derivative of $f(z)=\bar{z}$, if it exists. Give justification.

4. Show that if $Re(u)>0$ and $Re(v)>0$, then $log(uv) = logu + logv$.

UNIT–II

Answer Question No. **1** *and* ***any two*** *from the rest :*

4×3=12

1. Let $r = 2 + 4\cos\theta (0<\theta<2\pi)$ be a given curve.

If $I_1 = \int_r \frac{dz}{z-1}$ and $I_2 = \int_r \frac{dz}{z-3}$ find the relation between I_1 and I_2.

2. Let C be an arc of the circle of radius $|z|=2$ that lies in the first quadrant.

Without evaluating the integral, show that

$$\left|\int_C \frac{dz}{z^2-1}\right| \leq \frac{\pi}{3}.$$

3. Let C denote the positively oriented boundary of the square which sideline along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate—

$$\int_C \frac{e^{-2}}{\left(z-i\frac{\pi}{2}\right)} dz$$

4. State and prove the maximum-modulus principle.

UNIT–III

Answer Question No. **1** *and* ***any two*** *from the rest :*

4×3=12

1. Find the Maclaurin series expansion of the function

$$f(z) = \frac{z^3}{z^3+9}$$

What is its domain of validity ?

2. Find Taylor series representation of

$\frac{1}{1-z}$ in the neighbourhood of $z = -1$.

What is its domain of validity?

3. Represent the function $f(z) = \frac{1}{4z - z^2}$ by its Laurent series in the domain $0 < |z| < 4$.

4. Show that when $0 < |z-1| < 2$,

$$\frac{z}{(z-1)(z-3)} = -\frac{1}{2(z-1)} - 3\sum_{n=0}^{\infty} \frac{(z-1)^n}{2^{n+2}}.$$

UNIT–IV

Answer Question No. **1** *and* ***any two*** *from the rest* :

4×3=12

1. Using residue theorem, evaluate $\int_C \frac{\cosh z dz}{z(z^2+1)}$, where C is the circle $|z| = 2$ described in the positive sense.

2. Use theorem involving single residue, to evaluate the integral of

$$f(z) = \frac{z^3 e^{\frac{1}{z}}}{1+z^3}$$

around the positively oriented circle $|z| = 3$.

3. Use residues to find the Cauchy principal value of the improper integral

$$\int_0^{\infty} \frac{x^2 dx}{(x^2+9)(x^2+4)^2}.$$

4. *(i)* State argument principle. Find $\Delta_C \arg f(z)$,

where $f(z) = \frac{2(z-1)\cdot(z-2)}{(z-3)(z-3)^2}$ and $C: |z| = 4$.

(ii) State Rouche's theorem. Determine the number of zeroes, counting multiplicities, of the equation

$$z^4 + 3z^2 + 6 = 0$$

inside the circle $|z| = 2$.

UNIT–V

Answer Question No. **1** *and* ***any two*** *from the rest :*

4×3=12

1. Show that under the transformation $w = \sin z$, a line $x = c(\pi/2 < c < \pi)$ is mapped onto the righthand branch of a hyperbola. Further show that the mapping is one to one and that the upper and lower halves of the line are mapped onto the lower and upper halves, respectively.

2. Find the image of the semi-infinite strip $x > 0,\ 0 < y < 1$ where $w = i/z$.

   Sketch the strip and its image.

3. Find the bilinear transformation that maps the points $z_1 = \infty,\ z_2 = i,\ z_3 = 0$ onto the points $w_1 = 0,\ w_2 = i,\ w_3 = \infty$.

4. Explain the transformation $w = z^2$.
