Total No. of Printed Pages—6

2 SEM TDC MTMH (CBCS) C 3

2025

(May)

MATHEMATICS

(Core)

Paper : C–3

(Real Analysis)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

(*Marks* : 30)

1. (a) Define ε-neighbourhood of a point. 1

(b) State the completeness property of real numbers. 1

(c) Show that

$$\text{Sup}\left\{1-\frac{1}{n} : n \in \mathbb{N}\right\} = 1$$

2

(d) Define the following : 1+1=2

(i) Limit point of a set

(ii) Isolated point

(e) Find the infimum and supremum, if it exists for the set

$$A = \{x \in \mathbb{R} : 2x + 5 > 0\}$$ 2

(f) Prove that if x is a rational number and y is an irrational number, then $x + y$ is an irrational number. 3

(g) If x and y are any real numbers with $x < y$, then prove that there exists a rational number $r \in \mathbb{Q}$ such that $x < r < y$. 4

(h) State and prove the Bolzano-Weierstrass theorem for sets. 1+4=5

(i) Prove that the set of rational numbers in [0, 1] is countable. 5

Or

Prove that every subset of a countable set is countable.

(j) Prove that there does not exist a rational number r such that $r^2 = 2$. 5

Or

State and prove the Archimedean property of real numbers.

UNIT—II

(*Marks* : 30)

2. (a) Define real sequence and the range of a sequence. 1+1=2

(b) Write an example of a constant sequence. 1

(c) State True or False : 1

Every bounded sequence is convergent.

(d) Define the following : 1½+1½=3

(i) Cauchy sequence

(ii) Limit of a sequence

(e) Show that

$$\lim_{n \to \infty} \frac{3 + 2\sqrt{n}}{\sqrt{n}} = 2$$ 3

(f) Show that the sequence $\{S_n\}$, where

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

cannot converge. 4

Or

Show that the sequence $\{S_n\}$, where

$$S_n = \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \cdots + \frac{1}{\lfloor n}, \ \forall n \in \mathbb{N}$$

is convergent.

(g) Prove that every convergent sequence is bounded. 5

(h) State and prove the monotone convergence theorem. 1+4=5

(i) Prove that

$$\underset{n \to \infty}{\text{Lt}} (n^{1/n}) = 1$$

4

Or

Let $X = \{x_n\}$ be a sequence such that

$$\underset{n \to \infty}{\text{Lt}} \{x_n\} = x$$

Prove that

$$\underset{n \to \infty}{\text{Lt}} \{|x_n|\} = |x|$$

(j) State the monotone subsequence theorem. 2

UNIT—III

(*Marks* : 20)

3. *(a)* State the Cauchy criterion for convergence of a series. 1

(b) Define alternating series with an example. 1+1=2

(c) State Cauchy's root test. 2

(d) Show that the series

$$\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots$$

converges for $p > 0$. 3

(e) Show that the series

$$1 + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} + \cdots$$

is convergent. 4

Or

Show that the series

$$x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots$$

converges absolutely for all values of x.

(f) Discuss the convergence of a geometric series. 4

(g) Define absolute convergence. Show that if a series of real numbers is absolutely convergent, then it is convergent. 1+3=4

Or

Show that for any fixed value of x, the series

$$\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$$

is convergent. 4

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Total No. of Printed Pages—6

2 SEM TDC MTMH (CBCS) C 4

2025

(May)

MATHEMATICS

(Core)

Paper : C–4

(Differential Equations)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Is the differential equation $(2x-y+1)dx+(2y-x+1)dy=0$ exact? 1

(b) Find the integrating factor of the differential equation $\frac{dx}{dy}+Px=Q$, where P and Q are functions of y. 1

(c) Show that $y = e^{x}(\sin x + \cos x)$ is a solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$ 3

(d) Solve the following initial value problem : 2

$$\frac{dy}{dx} = e^{x+y},\ y(1) = 1$$

(e) Solve : 2

$$\frac{dy}{dx} = \frac{y - x^2}{x + y^2}$$

(f) Solve any *two* from the following : 3×2=6

(i) $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

(ii) $x\frac{dy}{dx} - 2y = 2x^4$

(iii) $(x + y)dx - xy\,dy = 0,\ y(1) = 2$

2. (a) What is compartmental model? 1

(b) Draw the input-output compartmental diagram for CO_2. 1

(c) Derive the formula for half-life of radioactive material. 2

(d) Answer any *two* from the following : 3×2=6

(i) Derive the differential equation of the Lake pollution model.

(ii) The volume of the lake is $28 \times 10^6\ m^3$, the average summer flow rate for the water into and out of the lake is $4 \times 10^6\ m^3$/month. How long will it take to reduce the pollution level 5% of its current level?

(iii) Write the assumptions for deriving the differential equation of drug assimilation into the blood model.

3. (a) Define linear combination of n functions. 1

(b) State the principle of superposition for homogeneous differential equation. 1

(c) Define Wronskian of two functions. 1

(d) Show that e^{2x} and e^{3x} are the solutions of the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

Also write the linear combinations of the above solution. 3

(e) Answer any *one* of the following : 4

(i) Show that two solutions $y_1(x)$ and $y_2(x)$ of the equation

$$a_0(x)\frac{d^2y}{dx^2}+a_1(x)\frac{dy}{dx}+a_2(x)y=0,\ a_0(x)\neq 0,\ x\in(a,b)$$

are linearly independent if and only if their Wronskian is not zero at some point $x_0 \in (a, b)$.

(ii) Given that e^{-x}, e^{3x} and e^{4x} are all solutions of

$$\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+5\frac{dy}{dx}+12y=0$$

Show that they are linearly independent and write the general solution.

4. Answer any *three* from the following : 5×3=15

(a) Solve :

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=x^3$$

(b) Solve :

$$\frac{d^2y}{dx^2}-y=x^2\cos x$$

(c) Solve the following by the method of undetermined coefficient :

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2$$

(d) Solve the following by the method of variation of parameters :

$$\frac{d^2y}{dx^2}+n^2y=\sec nx$$

5. (a) What do you mean by phase plane diagram? 1

(b) Write the word equation and differential equation for an epidemic model of influenza. 2

Or

Solve the system of differential equation $\frac{dx}{dt}=y,\ \frac{dy}{dt}=-x$ using separation of variable.

(c) Determine a compartment diagram and appropriate word equation for each of the two populations in the predator and prey model. 4

(d) Formulate the differential equation for model of battle. 3

Or

Sketch the phase-plane trajectory and determine the direction of trajectory of predator-prey model.

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