3 SEM PG (CBCS) MTH G 2

2025

(December)

MATHEMATICS

Paper : MTH G 2

(Mathematical Modelling)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

UNIT–I

1. Answer ***any three*** : 5×3=15

   (a) *(i)* Find the order and degree of the following D.E. : 2

$$\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^5 + y = x^2$$

(ii) Show that $y = a\cos(mx+b)$ is a solution of the D.E.

$$\frac{d^2y}{dx^2} + m^2y = 0$$ 3

(b) Solve :

(i) $y - x\left(\frac{dy}{dx}\right) = a\left(y^2 + \frac{dy}{dx}\right)$ 2

(ii) $\frac{dy}{dx} = (4x+y+1)^2 ; y(0) = 1$ 3

(c) Solve : $\left(x^3+y^3\right)dx - \left(x^2y+xy^2\right)dy = 0$ 5

(d) Solve :

(i) $\left(\frac{d^3y}{dx^3} - 8\right)y = 0$ 2

(ii) $\left(4D^2 - 12D + 9\right)y = 144e^{3x/2} ; D \equiv \frac{d}{dx}$ 3

UNIT–II

2. Answer ***any three*** : 5×3=15

(a) (i) In an archeological wooden specimen, only 25% of original radio-active carbon 14 is present. When was it made ? (half-life of carbon 14 is 5568 years). 2

(ii) Develop a mathematical model to find the velocity of an object falling under gravity in a resisting medium. 3

(b) Develop a model to show that equilateral triangles have maximum area for a given perimeter.

(c) Derive an expression for the linear growth of a population and hence find the half-life and doubling periods of the population.

(d) Explain the logistic law of population growth.

UNIT–III

3. Answer the following : 1×3=3

(a) **Define :** Graph, Degree of vertex.

(b) **Prove or disprove :** "The number of vertices of odd degree in a graph is always even".

(c) When *two* graphs are said to be isomorphic ?

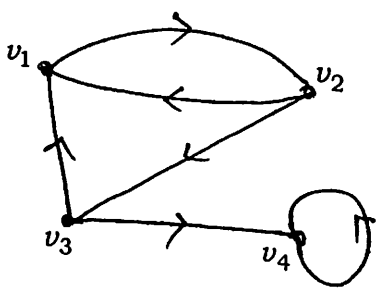
4. Answer ***any three*** : 2×3=6

(a) What is Eulerian graph ? Is K_4 Eulerian ? 1+1=2

(b) Show that if a graph has exactly two vertices of odd degree, there must be a path joining them.

(c) What is a simple graph ? Find the maximum number of edges simple graph can have with 11 vertices. 1+1=2

(d) What is a directed graph ? Find in-degree and out-degree of all vertices in the following digraph : 1+1=2

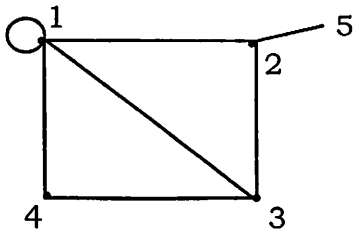


5. Answer ***any two*** : 3×2=6

(a) Show that a tree with n vertices has $n-1$ edges.

(b) State and prove a necessary and sufficient condition for a connected graph to be Eulerian.

(c) Define adjacency matrix. Find adjacency matrix of the following graph : 1+1+1=3



Also, write *any two* properties of adjacency matrix.

UNIT–IV

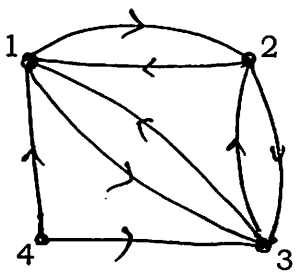
6. Answer ***any three*** : 5×3=15

(a) (i) Describe *two* situations that can be modelled through graphs. 2

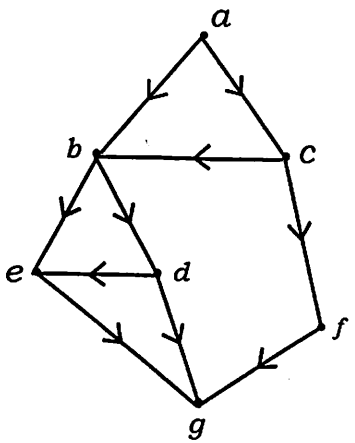
(ii) What is a clique in a socio-psychological group ? Explain a use of directed graph in detection of cliques. 1+2=3

(b) (i) What is a signed graph ? Provide an example of a balanced signed graph. 1+1=2

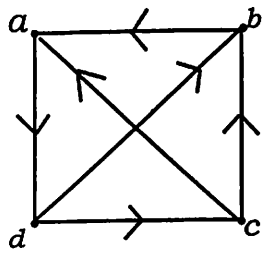
(ii) Find the number of walks of length *two* between any pairs of vertices in the following graph. 3



(c) (i) Find Harary measure $(h(x))$ for each individual in the following organisational digraph. 2



(ii) Find the fractions of messages that pass through each individual in the communication network given below : 3



(d) Discuss Weighted Digraphs and Markov chains.