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3 SEM TDC GEMT (CBCS) GE 3 (A/B/C)

2025

(Nov/Dec)

MATHEMATICS

(Generic Elective)

Paper : GE–3

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : GE–3A

(Real Analysis)

1. *(a)* Define countable set. 1

(b) Is the set $S = \{2n : n \in \mathbb{N}\}$ denumerable? Justify. 3

(c) Prove that for a set supremum and infimum (if exists) are unique. 4

Or

Find the supremum or/and infinimum of the set $X = \{x \in I / x^2 \leq 36\}$.

(d) State and prove the Archimedean property for real numbers. 4

(e) Prove that the set of natural numbers N is not bounded above. 4

Or

Prove that the real number field $\mathbb{R}$ is an Archimedean field.

(f) State and prove the Bolzano-Weierstrass theorem for sets. 4

2. (a) State true or false : 1

The sequence $\left\{\frac{1}{n}\right\}$ is convergent.

(b) Every convergent sequence is bounded. Is the converse true? Justify. 3

(c) Show that the sequence $\left\{\frac{n+1}{n}\right\}$ converges to 1. 3

Or

Show that every convergent sequence is bounded.

(d) State and prove the Bolzano-Weierstrass theorem for sequences. 5

(Continued)

Or

Define Cauchy sequence. Prove that every sequence is convergent if and only if it is Cauchy sequence.

(e) Show that the sequence $\left\{\frac{3n+7}{4n+8}\right\}$ is monotonic. Is it monotonically increasing or decreasing? 3

Or

Prove that the sequence $\left\{x_n = \frac{n+1}{n}\right\}$ is a Cauchy sequence.

(f) State and prove Cauchy's general principle of convergence of sequence. 5

3. (a) Define infinite series. 1

(b) Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$ 5

Or

Prove that the positive term geometric series $1+r+r^2+r^3+\ldots$ converges for $r<1$ and diverges to ∞ for $r \geq 1$.

(c) Prove that the series Σu_n where $u_n = -n$ diverges to $-\infty$. 2

(Turn Over)

Or

Show that the series $1^2+2^2+3^2+\ldots$ diverges to ∞.

(d) State Cauchy's general principle of convergence of an infinite series. 2

Or

Test the convergence of the series $1+2+3+\ldots$

(e) Test the convergence of any *two* of the following : 5×2=10

(i) $\Sigma\frac{1}{(\log n)^n}$

(ii) $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\frac{4^4}{5^5}+\cdots$

(iii) $\Sigma[(n+1)^{\frac{1}{2}}-n^{\frac{1}{2}}]$

(iv) $\sum_{n=1}^{\infty}\frac{n^2-1}{n^2+1}$

4. *(a)* Define alternating series. Show that the series $\Sigma(-1)^{n-1}\frac{1}{x+(n-1)a}$, $x>0$, $a>0$ is convergent. 1+4=5

(Continued)

Or

Define uniform convergence of sequence. Show that the sequence $\{f_n\}$, where $f_n(x)=\frac{x}{1+nx^2}$ is uniformly convergent on any closed interval I. 5

(b) Give an example of conditional convergent series. 1

(c) Show that the series

$$\sum_{n=1}^{\infty}\frac{a_n x^n}{1+x^{2n}}$$

is uniformly convergent for all real x if $\sum_{n=1}^{\infty}a_n$ is absolutely convergent. 5

Or

Let $\{f_n\}$ be a sequence of functions such that $\underset{n\to\infty}{\text{Lt}}\, f_n(x)=f(x)$, $x\in[a,\ b]$ and let $M_n=\sup_{x\in[a,\ b]}|f_n(x)-f(x)|$. Prove that $f_n\to f$ uniformly on $[a,\ b]$ if and only if $M_n\to 0$ as $n\to\infty$.

(d) If a power series $\Sigma a_n x^n$ converges for $x=x_0$, then prove that it is absolutely convergent for every $x=x_1$ when $|x_1|<|x_0|$. 5

(Turn Over)

(e) Find the radius of convergence of the following series (any *one*) : 4

(i) $1+\frac{3}{5}x+\frac{3\cdot5}{5\cdot10}x^2+\ldots$

(ii) $\Sigma(4+3i)^n 2^n$.

(Continued)

Paper : GE–3B

(Cryptography and Network Security)

1. (a) Write true or false : 1

"Encryption ensures data confidentiality."

(b) Choose the correct option : 1

In a digital signature scheme, the sender signs the message using his

(i) public key

(ii) private key

(iii) session key

(iv) symmetric key

(c) What is the one-way property of a hash function? 1

(d) Write true or false : 1

"Symmetric encryption is faster than asymmetric encryption."

(e) Define 'confidentiality' and explain its role in securing communication. 2

(f) Explain the working of the Diffie-Hellman key exchange algorithm. 10

Or

Describe AES algorithm.

(Turn Over)

(g) Illustrate how HMAC works with an example. 8

Or

Briefly explain MD5 algorithm.

(h) Define two secure protocols used for web traffic encryption. 2

(i) Describe how SHA-256 algorithm works and explain its importance in data integrity. 8

Or

Explain the concept of message authentication code (MAC) and how it is different from digital signature.

2. (a) Describe the importance of certificate revocation in public key infrastructure (PKI). 2

(b) What is a buffer overflow attack and how can it be prevented? 8

Or

What is a Denial of Service (DoS) attack, and how can it be mitigated?

Or

What is a 'man-in-the-middle' attack, and how can it be prevented?

(Continued)

(c) Explain any *three* of the following : 3×3=9

(i) Packet sniffing

(ii) Brute force attack

(iii) Digital signature

(iv) Key management

(v) Diffusion and confusion

(d) Briefly explain how intrusion detection systems (IDS) work and how they help in network security. 5

3. (a) What is the purpose of the authentication header (AH) in IPSec? 2

(b) Describe the IPSec architecture. 8

Or

Briefly explain about encapsulating security payload.

(c) Explain secure socket layer (SSL). 7

Or

Explain UDP session hijacking in brief.

(d) What is e-mail security? Explain the technique for e-mail security. 1+4=5

Or

Differentiate between tunnel mode and transport mode. 5

(Turn Over)

Paper : GE–3C

(Information Security)

1. Answer the following questions : 2×10=20

(a) What type of security was dominant in the early years of computing?

(b) Differentiate between encryption and hashing.

(c) What does two-factor authentication mean?

(d) What is phishing?

(e) Mention two uses of digital signature.

(f) What are plain text and ciphertext in cryptography?

(g) What is Denial of Service (DoS) attack?

(h) What is firewall used for?

(i) Differentiate between threats and attacks.

(j) Mention one difference between symmetric and asymmetric encryption.

2. Answer any *four* of the following : 3×4=12

(a) Explain the three components of the CIA triad with an example for each.

(Continued)

(b) Why is communication privacy important?

(c) What is orange book in information security?

(d) Explain the difference between virus, worm and trojan horse.

(e) What is a zero-day vulnerability? Give a real-world example.

3. Briefly explain encryption and decryption with a suitable example. 6

4. Explain how digital signatures provide authentication, integrity, and non-repudiation. 6

Or

What is Ransomware? Explain its working mechanism and methods of prevention. 2+4=6

5. Briefly explain the principles of information security. 5

6. What is an intrusion detection system? Explain the working principle of a network based intrusion detection system. 2+3=5

Or

What are the challenges in designing an efficient intrusion detection system? 5

(Turn Over)

7. Discuss the different types of information system audits. 6

Or

Why are system calls important for intrusion detection? Explain how system call monitoring helps in behaviour-based intrusion detection. 2+4=6

8. Answer any *two* of the following : 10×2=20

   (a) What is digital signature? Briefly explain the working principle of digital signature. 2+8=10

   (b) Explain five different types of information security.

   (c) What are the important components of an information system? Explain.

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