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**3 SEM TDC MTMH (CBCS) C 5**

**2 0 2 4**

( Nov/Dec )

**MATHEMATICS**

( Core )

Paper : C-5

**( Theory of Real Functions )**

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

1. (a) Define cluster point of a set. 1  
(b) Write the sequential criterion for limits. 2  
(c) Using the definition of limit, evaluate

$$\lim_{x \rightarrow 3} \frac{2x+3}{4x-9} \quad 3$$

Or

Prove that  $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$  does not exist in  
the set of real numbers.

2. (a) What do you mean by one-sided limits of a function  $f$  at a point  $C$ ? 1

(b) Discuss the kind of discontinuity of the function

$$f(x) = \begin{cases} \frac{x-|x|}{x}, & \text{when } x \neq 0 \\ 2, & \text{when } x = 0 \end{cases} \quad 2$$

(c) State and prove squeeze theorem. 3

Or

Apply squeeze theorem to show that

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

(d) Show that the function  $f(x)$  defined on  $\mathbb{R}$ , the set of real numbers, by

$$f(x) = \begin{cases} x, & \text{when } x \text{ is irrational} \\ -x, & \text{when } x \text{ is rational} \end{cases}$$

is continuous only at  $x=0$ . 4

Or

Let  $A, B \in \mathbb{R}$ , let  $f: A \rightarrow \mathbb{R}$  be continuous on  $A$  and let  $g: B \rightarrow \mathbb{R}$  be continuous on  $B$ . If  $f(A) \subseteq B$ , then prove that the composite function  $g \circ f: A \rightarrow \mathbb{R}$  is continuous on  $A$ .

3. (a) State the preservation of intervals theorem. 1

(b) State and prove location of roots theorem. 4

(c) Prove that if a function is continuous in a closed interval, then it is bounded therein. 4

4. (a) A function continuous on a closed interval may not be uniformly continuous on that interval. State true or false. 1

(b) Prove that  $\sin x$  is uniformly continuous on  $[0, \infty]$ . 4

Or

Prove that

$$f(x) = \begin{cases} \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is not uniformly continuous on  $[0, \infty]$ .

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5. (a) Continuity is the sufficient condition for derivability of a function at a point. State true or false. 1
- (b) Write the statement of interior extremum theorem. 2
- (c) Prove that if  $f: I \rightarrow \mathbb{R}$  has a derivative at  $C \in I$ , then  $f$  is continuous at  $C$ . 3

Or

Examine the differentiability of the function  $|x|$  at  $x = 0$ .

6. (a) A convex function on an open interval is necessarily continuous. State true or false. 1
- (b) Give an example to show that a convex function need not be differentiable at every point of its domain. 2
- (c) Write the geometrical interpretation of Rolle's theorem. 3

Or

Show that the Rolle's theorem is not applicable to the function  $f(x) = \tan x$  in the interval  $(0, \pi)$ .

- (d) State and prove Darboux's theorem. 4

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7. (a) If  $\phi'(x) = \psi'(x)$  in an interval, then prove that  $\phi(x)$  and  $\psi(x)$  differ by a constant in that interval. 2

- (b) Prove that

$$\lim_{h \rightarrow 0} \frac{f(a+h) - 2f(a) + f(a-h)}{h^2} = f''(a)$$

provided  $f''(x)$  is continuous. 3

Or

If  $f(x)$  and  $\phi(x)$  are continuous in  $a \leq x \leq b$  and differentiable in  $a < x < b$  such that  $f'(x)$  and  $\phi'(x)$  never vanish for the same value of  $x$ , then show that

$$\frac{f(\xi) - f(a)}{\phi(b) - \phi(\xi)} = \frac{f'(\xi)}{\phi'(\xi)}$$

where  $a < \xi < b$ .

- (c) Show that  $\log(1+x)'$  lies between  $x - \frac{x^2}{2}$  and  $x - \frac{x^2}{2(1+x)}$ ,  $\forall x > 0$ . 4

8. (a) Write the condition of validity of expansion of  $e^x$  in powers of  $x$  in infinite series. 1

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( 6 )

(b) If  $f(x) = x^2$ ,  $\phi(x) = x$ , then find a value of  $\xi$  in terms of  $a$  and  $b$  in Cauchy's mean value theorem. 2

(c) Show that the Cauchy's remainder after  $n$  terms in the expansion of  $\log(1+x)$  in powers of  $x$  is

$$(-1)^{n-1} \frac{x^n}{1+\theta x} \left( \frac{1-\theta}{1+\theta x} \right)^{n-1}, \quad 0 < \theta < 1 \quad 4$$

(d) State and prove Cauchy's mean value theorem. 5

Or

Deduce Taylor's theorem from Cauchy's mean value theorem.

9. (a) What do you mean by a relative extremum of a function at a point? 1

(b) Show that  $x^3 - 6x^2 + 12x - 3$  is neither a maximum nor a minimum when  $x = 2$ . 2

(c) Give an example of a convex function which is not differentiable at a point. 2

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(d) Expand the following functions in powers of  $x$  in infinite series stating the conditions under which the expansion is valid (any two) :  $4 \times 2 = 8$

(i)  $\sin x$

(ii)  $(1+x)^n$

(iii)  $\frac{1}{1+x}$

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**3 SEM TDC MTMH (CBCS) C 7**

**2 0 2 4**

( Nov/Dec )

**MATHEMATICS**

( Core )

Paper : C-7

**( PDE and Systems of ODE )**

Full Marks : 60

Pass Marks : 24

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

1. (a) Define general integral of a differential equation. 1
- (b) Give an example of Lagrange's linear equation. 1
- (c) Find the complete solution of  $pq = 1$ . 1
- (d) Form the PDE of all spheres of radius  $a$  having centre in the  $xy$ -plane. 5
- Or
- Solve  $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ .
- (e) Find the equation of surface satisfying  $4yzp + q + 2y = 0$  and passing through  $y^2 + z^2 = 1$  and  $x + z = 2$ . 5
- Or
- Solve  $yp = 2yx + \log q$ .

( 2 )

2. (a) Find Jacobi's auxiliary equation for  $p_1x_1 + p_2x_2 - p_3^2 = 0$ . 2

(b) Solve any one of the following : 4

(i)  $2xz - px^2 - 2qxy + pq = 0$

(ii)  $(p^2 + q^2)y - qz = 0$

(iii)  $px + qy = z(1 + pq)^{\frac{1}{2}}$

(c) Find complete integral of  $p_1^3 + p_2^2 + p_3 - 1 = 0$  6  
Or

Solve the boundary value problem  $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$  with  $u(0, y) = 8e^{-3y}$  by the method of separation of variables.

3. (a) Write the wave equation. 1

(b) Classify the equation  $u_{xx} + u_{yy} + u_{zz} + u_{yz} + u_{zy} = 0$  2

(c) Classify the operator  $\frac{\partial^2 u}{\partial x^2} + t \frac{\partial^2 u}{\partial x \partial t} + x \frac{\partial^2 u}{\partial t^2}$  2

(d) Reduce the equation  $y(x+y)(r-s) - xp - yq - z = 0$  to canonical form. 7  
Or

Derive the one-dimensional heat equation.

4. (a) Write one assumption of vibrating string problem. 1

( 3 )

(b) Write the general form of two-dimensional heat equation. 1

(c) Solve

$$\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0$$

using the method of separation of variables. 6

Or

$$\text{Solve } \frac{\partial^2 u}{\partial x^2} = k^2 \left( \frac{\partial u}{\partial t} \right)$$

when

$$u(0, t) = u(l, t) = 0,$$

$$u(x, 0) = \sin \frac{\pi x}{l}$$

5. (a) Write the differential equation

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = 0$$

using  $D$ . 1

(b) Transform

$$\frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} - 24x = 0$$

into a system of first-order differential equation. 2

(c) If  $x = f_1(t)$ ,  $x = f_2(t)$  and  $y = g_1(t)$ ,  $y = g_2(t)$  be the solution of

$$\frac{dx}{dt} = a_{11}(t)x + a_{12}(t)y$$

$$\frac{dy}{dt} = a_{21}(t)x + a_{22}(t)y$$

then show that

$$x = c_1 f_1(t) + c_2 f_2(t)$$

$$\text{and } y = c_1 g_1(t) + c_2 g_2(t)$$

is also the solution of the system. 2

(d) Solve the system

$$2 \frac{dx}{dt} - 2 \frac{dy}{dt} - 3x = t$$

$$2 \frac{dx}{dt} + 2 \frac{dy}{dt} + 3x + 8y = 2 \quad 6$$

Or

Find the general solution of the linear system

$$\frac{dx}{dt} = x - 3y$$

$$\frac{dy}{dt} = 3x + y$$

(e) For initial value problem

$$\frac{dy}{dx} + 2y = 2 - e^{-4x}, \quad y(0) = 1$$

use Euler's method to find the solution for  $x = 0.1$  and  $0.2$ . 4

Or

Describe Picard method of successive approximation.

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