Total No. of Printed Pages—19

4 SEM TDC GEMT (CBCS) 4.1/4.2/4.3

2025

(May/June)

MATHEMATICS

(Generic Elective)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

All symbols have their usual meanings

Paper : GE-4.1

(Algebra)

UNIT—1

1. Answer the following questions :

   (a) Fill in the blank : 1

   The number of symmetries of a rectangle is _____.

(b) State True or False : 1

The set πQ is a group under usual addition.

(c) Show that in a group G, $(a^{-1})^{-1} = a$ for any $a \in G$. 2

(d) Find the inverse of the element $-j$ in the group of quaternions. 2

(e) Prove that if $(ab)^2 = a^2b^2$ in a group G, then $ab = ba$. 3

(f) Let G be a group such that the square of any element is unity. Show that G is Abelian. 3

(g) Describe the symmetries of a square. 4

Or

Describe the circle group.

(h) Prove that the set $\{1, 2, \ldots, n-1\}$ is a group under multiplication if and only if n is prime. 4

Or

Prove that the set of all 3×3 matrices with real entries of the form

$$\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

is a group under matrix multiplication.

UNIT—2

2. Answer the following questions :

(a) State Lagrange's theorem. 1

(b) State true or false : 1

Subgroup of a cyclic group is cyclic.

(c) Let $H = \{(1), (12)(34), (13)(24), (14)(23)\}$. How many left cosets of H in S_4 are there? 1

(d) Show that the centre of a group is an Abelian subgroup. 2

(e) Let G be a group of order 60. What are the possible orders for the subgroups of G? Justify. 2

(f) Consider the subgroup $H = \{\pm 1, \pm i\}$ of the group of quaternions. Find any three left cosets of H. 3

(g) Suppose that $|G| = pq$, where p and q are primes. Prove that every proper subgroup of G is cyclic. 3

(h) Let H be a subgroup of a group G. Show that if index of H in G is 2, then H is normal in G. 3

(i) Consider $H = \{1, 11\}$ of $U(30)$. Find the quotient group $U(30)/H$. 4

(j) $$H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in R, ad \neq 0 \right\}$$

Examine whether H is a normal subgroup of $GL(2, R)$. 5

Or

Prove that the factor group of an Abelian group is Abelian.

(k) Show the intersection of two normal subgroups is also a normal subgroup. 5

Or

Let G be a group and let G' be the commutator subgroup of G. Prove that

(i) G' is normal in G;

(ii) if H is a subgroup of G and $H \supseteq G'$, then H is normal in G.

UNIT—3

3. Answer the following questions :

(a) State True or False : 1+1=2

(i) Every ring has a multiplicative inverse.

(ii) Every element in a ring has an additive inverse.

(b) Show that the polynomial $2x+1$ in $Z_4[x]$ has a multiplicative inverse. 2

(c) Justify that the ring of all 2×2 matrices over reals under usual addition and multiplication of matrices is a non-commutative ring. 2

(d) List all polynomials of degree 2 in $Z_2[x]$. 3

(e) Show that the non-zero elements of a field form a group under multiplication. 4

(f) Show that the ring $Z[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in Z\}$ is an integral domain. 4

(g) Consider the equation $x^2 - 5x + 6 = 0$. Find all solutions of this equations in Z_8. 3

(h) Let $S = \{a + ib\} a, b \in Z, b$ is even. Show that S is a subring of $Z[i]$ but not an ideal of $Z[i]$. 5

Or

Prove that the intersection of any set of ideals of a ring is an ideal.

Paper : GE–4.2

(Application of Algebra)

1. তলৰ যি কোনো দুটা প্ৰশ্নৰ উত্তৰ দিয়া : 6×2=12

Answer any *two* of the following questions :

(a) প্ৰমাণ কৰা যে, (m, b, r, k, λ) প্ৰাচলৰ সৈতে এটা BIBD সমমীতিয় হয় যদি আৰু কেৱল যদিহে $r = k$ হয়।

Prove that a BIBD with parameters (m, b, r, k, λ) is symmetric if and only if $r = k$.

(b) ধৰাহওক, $p > 2$ আৰু p এটা মৌলিক সংখ্যা। তেন্তে প্ৰমাণ কৰা যে তাত $(p-1)/2$টা দ্বিঘাত ৰেচিডিউ মডুল' p থাকে আৰু

$$Q_p = \left\{res_p(n^2) \mid 1 \le n \le \frac{p-1}{2}\right\}$$

Let p be a prime number greater than 2. Then prove that there are $(p-1)/2$ quadratic residues modulo p, and

$$Q_p = \left\{res_p(n^2) \mid 1 \le n \le \frac{p-1}{2}\right\}$$

(c) ধৰাহওক, F; $6t+1$ মাত্ৰা (order)-ৰ সীমিত ফিল্ড, আৰু a হৈছে F-ৰ এটা প্ৰিমিটিভ মৌল আৰু ধৰাহওক $S_i = \{a^i, a^{2t+i}, a^{4t+i}\}, \quad i = 0, 1, \ldots, t-1$. তেন্তে দেখুওৱা যে $S_0, \ldots, S_{t-1}$ সংহতিবোৰে $(6t+1, 3, 1)$ পাৰ্থক্য সংহতি পৰিয়ালৰ যোগাত্মক গ্ৰুপ F-ৰ এটা t-ফ'ল্ড গঠন কৰে।

Let F be a finite field of order $6t+1$ and let a be a primitive element in F. Let $S_i = \{a^i, a^{2t+i}, a^{4t+i}\}, \quad i = 0, 1, \ldots, t-1$. Then show that the sets $S_0, \ldots, S_{t-1}$ form a t-fold $(6t+1, 3, 1)$ difference set family in the additive group F.

2. BIBD-ৰ ইন্সিডেন্স মেট্ৰিক্স-ৰ ওপৰত এটা চমু টোকা লিখা। 4

Write a short note on incidence matrix of a BIBD.

অথবা / Or

ধৰাহওক, $G = Z_7$ অখণ্ড সংখ্যা মডুল' 7-ৰ এটা যোগাত্মক গ্ৰুপ, আৰু $S = \{1, 2, 4\}$. দেখুওৱা যে, S হৈছে G-ৰ এটা পাৰ্থক্য সংহতি, আৰু ইয়াৰ প্ৰাচলবোৰ নিৰ্ণয় কৰা।

Let $G = Z_7$ be the additive group of integers modulo 7, and $S = \{1, 2, 4\}$. Show that S is a difference set in G, and find its parameters.

3. তলৰ যি কোনো দুটা প্ৰশ্নৰ উত্তৰ দিয়া : 6×2=12

Answer any *two* of the following questions :

(a) Parity-check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$ৰ

সৈতে দ্বৈত ৰৈখিক ক'ড C নিৰ্ণয় কৰা আৰু C-ৰ generator মেট্ৰিক্স G লিখা। লগতে Dual ক'ড $C^{\perp}$-ৰ মান নিৰ্ণয় কৰা।

Find the binary linear code C with parity-check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

and write a generator matrix G of C. Also find the dual code $C^{\perp}$.

(b) জেনেৰেটৰ মেট্ৰিক্স G-ৰ সৈতে দ্বৈত ক'ডৰ বাবে উন্নত সজ্জা লিখা আৰু 01111 ভেক্টৰটো ডিক'ড কৰা, য'ত

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Write a standard array for the binary code with the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

and decode the received vector 01111.

(c) প্ৰমাণ কৰা যে, $F[x]_n$ -ৰ উপসংহতি C এটা চাইক্লিক ক'ড হ'ব যদি আৰু কেৱল যদিহে C, $F[x]_n$ ৰিং-ৰ এটা আদৰ্শ হয়।

Prove that a subset C of $F[x]_n$ is a cyclic code if and only if C is an ideal of the ring $F[x]_n$.

4. দেখুওৱা যে, এটা দ্বৈত ক'ড (7, 16, 3) (যদিহে থাকে) এটা নিখুঁত ক'ড। 4

Show that a binary code (7, 16, 3) (if it exists) is perfect.

5. (a) দেখুওৱা যে

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 5 & 1 & 6 & 4 & 2 & 3 \end{pmatrix}$$

আৰু

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 1 & 6 & 2 & 3 & 7 & 5 \end{pmatrix}$$

বিন্যাস দুটাৰ গঠন চাইক্লিক আৰু একে। যদি $\beta = \sigma\alpha\sigma^{-1}$ হয়, তেন্তে σ -ৰ মান নিৰ্ণয় কৰা। 4

Show that the permutations

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 5 & 1 & 6 & 4 & 2 & 3 \end{pmatrix}$$

and

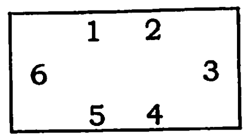
$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 1 & 6 & 2 & 3 & 7 & 5 \end{pmatrix}$$

have the same cyclic structure. Find σ such that $\beta = \sigma\alpha\sigma^{-1}$.

(b) অসমৰূপী গ্ৰাফৰ জেনেৰেটিং ফলনৰ ওপৰত এটা চমু টোকা লিখা। 4

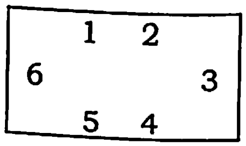
Write a short note on generating functions for non-isomorphic graph.

(c) এখন আয়তাকাৰ ডাইনিং টেবুলত 6 জন মানুহ এনেভাবে বহি আছে যাতে, দুজন টেবুলৰ দীঘল দৈৰ্ঘ্যফালে আৰু বাকী কেইজন টেবুলৰ চুটি দৈৰ্ঘ্যফালে মুখামুখিকৈ। m-টা ৰংৰ নেপকিনৰ পৰা তেওঁলোকক দিয়া হ'ল। তেওঁলোকৰ মাজত সকলো সম্ভৱ ৰংৰ নেপকিন বিতৰণৰ সকীয়া আৰ্হিৰ সংখ্যা বিচাৰি উলিওৱা : 8



A rectangular dining table seats six persons, two along each longer side and one on each shorter side. A colored napkin, having one of m given colors, is placed for each person.

Find the number of distinct patterns among all possible color assignments :



অথবা / Or

পলিয়া-ৰ উপপাদ্যটো উল্লেখ আৰু প্ৰমাণ কৰা।

State and prove Polya's theorem.

6. *(a)* ধৰাহওক, A আৰু B দুটা শূন্যক মেট্ৰিক্স যাৰ মাত্ৰা একে। যদি $AB = BA$, তেন্তে প্ৰমাণ কৰা যে, $A+B$ এটা শূন্যক মেট্ৰিক্স। 4

Let A and B be nilpotent matrices of the same size. If A and B commute, then show that $A+B$ is nilpotent.

(b) $n \times n$ নিৰ্ণায়কৰ মান নিৰ্ণয় কৰা : 4

Compute the $n \times n$ determinant :

$$\begin{vmatrix} 0 & 1 & & & & & 0 \\ 1 & 0 & 1 & & & & \\ & 1 & 0 & 1 & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & \cdot & \cdot & \cdot \\ 0 & & & & 1 & 0 & 1 \\ & & & & & 1 & 0 \end{vmatrix}$$

(Continued)

(c) Frobenius-König-ৰ উপপাদ্যটো উল্লেখ আৰু প্ৰমাণ কৰা। 8

State and prove Frobenius-König theorem.

অথবা / Or

যদি A এটা $m \times m$ হাদামাৰ্ড মেট্ৰিক্স হয় যাৰ J_n হৈছে এটা উপ-মেট্ৰিক্স, তেন্তে প্ৰমাণ কৰা যে $m \geq n^2$. (J_n হৈছে $m \times m$ মেট্ৰিক্স যাৰ প্ৰত্যেক মৌলবোৰ সমান আৰু সেইবোৰ সকলো 1.)

If A is an m-square Hadamard matrix that contains a J_n as a submatrix, then prove that $m \geq n^2$. (J_n denotes the m-square matrix whose entries are all equal to 1.)

7. যি কোনো দুটা প্ৰশ্নৰ উত্তৰ দিয়া : 8×2=16

Answer any *two* of the following questions :

(a) 3×2 মেট্ৰিক্স $A = \begin{bmatrix} 2 & 3 \\ 0 & 4 \\ 0 & 1 \end{bmatrix}$-ৰ আনুমানিক বিপৰীত মেট্ৰিক্সটো উলিওৱা।

(Turn Over)

Find the approximate inverse of the 3×2 matrix $A = \begin{bmatrix} 2 & 3 \\ 0 & 4 \\ 0 & 1 \end{bmatrix}$.

(b) ধৰাহওক, $A = LDU$ তলৰ তিনিটা মেট্ৰিক্সৰ গুণফল

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \\ 2 & 0 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 3 & 3 & 3 & 3 \\ 0 & 0 & 1 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 6 \end{bmatrix}$$

সমাধান কৰা, $LDU\ x = y$ য'ত y-ৰ মান

$$\begin{bmatrix} 2 \\ 9 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 9 \\ 6 \\ 4 \end{bmatrix}$$ হয়।

Let $A = LDU$ be the product of

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \\ 2 & 0 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 3 & 3 & 3 & 3 \\ 0 & 0 & 1 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 6 \end{bmatrix}$$

Solve $LDU\ x = y$ for the values

$$\begin{bmatrix} 2 \\ 9 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 9 \\ 6 \\ 4 \end{bmatrix}$$ for y.

(c) তলৰ মেট্ৰিক্সটো ৰ'-বিদিউসদ এশ্বিলন ফৰ্মলৈ নিবলৈ ৰ'-বিদাকশ্যন এলগ'ৰিথম ব্যৱহাৰ কৰা :

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

Use row-reduction algorithm to reduce the following matrix into row-reduced echelon form :

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

Paper : GE–4.3

(Combinatorial Mathematics)

1. *(a)* Find ${}^8P_3$. 1

(b) Write the principle of exclusion. 1

(c) A girl has 5 pencils of different colours. In how many ways she can arrange them? 2

(d) Find how many 2-digit numbers can be formed by using first 4 prime numbers. 2

(e) From a team of 14 boys, find how many football teams can be formed. 2

(f) Show that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$, if $1 \le r \le n$. 4

Or

Find the number of distinguishable words that can be formed from the letters of VACANT.

2. *(a)* Write the principle of pigeonhole. 1

(b) State true or false : 1

If there are more than m objects and there are m boxes, then there will be at least 1 box with no object.

(c) Find how many integers between 1 and 250 are—

(i) divisible by 3;

(ii) divisible by 3 and 7. 2+2=4

(d) Let A, B are finite sets. Show that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ 4

Or

Find the number of integer solutions of $x_1 + x_2 + x_3 = 24$, such that $1 \le x_1 \le 5$, $12 \le x_2 \le 18$, $-1 \le x_3 \le 12$.

3. *(a)* Write the generating function for 1, 1, 1, 1, ... 1

(b) Define a generating function. 2

(c) Find the co-efficient of x^4 in $(1-x)^{-2}$. 4

(d) A recursively defined sequence $a_n = 3a_{n-1} - 1$, $\forall\, n \ge 1$ and $a_0 = 2$. Find an explicit formula for a_n. 5

Or

Determine the set of integers n for which $n^2 + 19n + 92$ is a square.

4. Answer any *two* of the following questions : $5\times2=10$

(a) Find the number of binary sequences of length n having no 11.

(b) Prove that there exist $2^n - n$ numbers that have n digits made up only of numbers 1 and 2 and contain each digit at least once.

(c) If $n+1$ integers are chosen, show that there exist two integers whose difference is divisible by n, where n is a positive integer.

5. (a) Write the number of portions of 6. 2

(b) Determine how many integers between 1 and 60 are divisible by at least one of 2, 3 and 5. 5

(c) Find the number of integers between 1 and 10000 that are neither perfect squares nor perfect cubes. 5

Or

Let numbers 1 to 20 are placed in any order around a circle. Show that the sum of some 3 consecutive numbers must be at least 32.

6. (a) Write the number of ways to arrange n distinct objects in a circle. 1

(b) Find the number of arrangements of any 3 letters from the 11 letters of the word COMBINATION. 2

(c) Find the number of ways to arrange $n \geq 3$ differently coloured beads in a necklace. 4

(d) Find the number of different necklace that contain four red and three blue beads. 5

7. (a) Define a combinatorial design. 1

(b) Write one property of uniform design. 1

(c) Write an example of Latin square of order 3. 2

(d) Answer any *two* of the following : $4\times2=8$

(i) Prove that interchanging two rows of a Latin square produce a Latin square.

(ii) Show that there is no BIBD (balanced incomplete block design) with parameters $b=12$, $k=4$, $v=16$ and $r=3$ (λ not specified).

(iii) Determine the cycle index of the dihedral group D_4.

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