4 SEM TDC MTMH (CBCS) C 8

2025

(May/June)

MATHEMATICS

(Core)

Paper : C–8

(Numerical Methods)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

Use of scientific calculator is allowed

1. *(a)* State true or false : 1

An exact number may be regarded as an approximate number with error zero.

(b) Write an algorithm to find the root of a linear equation. 2

(c) Define relative error and absolute error. 1+1=2

2. *(a)* State true or false : 1

Bisection method is always convergent.

(b) Describe secant method for solving an algebraic equation. 4

Or

Find a real root of the equation

$$x^3 - 2x - 5 = 0$$

by secant method correct up to three decimal places.

(c) Describe the geometrical interpretation of Newton-Raphson method. 5

Or

Determine the real root of

$$2x = \cos x + 3$$

by using iteration method correct up to three decimal places.

3. (a) Describe Gauss-Seidel method for the solution of system of linear equations. 5

Or

Solve by Gaussian elimination method

$$x + y - z = 2$$
$$2x + 3y + 5z = -3$$
$$3x + 2y - 3z = 6$$

(b) Solve by Gauss-Jordan method

$$5x - 2y + 3z = -1$$
$$-3x + 9y + z = 2$$
$$2x - y - 7z = 3$$

5

Or

Find the solution of the following system of equations by Gauss-Jacobi method :

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

4. (a) Define interpolation. 1

(b) Evaluate

$$\Delta^3(1-x)(1-2x)(1-3x)$$

if $h = 1$. 2

(c) Construct forward difference table for the following values : 2

x	0	5	10	15	20	25
y	5	9	12	16	22	30

(d) Deduce Lagrange's interpolation formula. 5

Or

The population of a town is as follows :

Year	x	1971	1981	1991	2001	2011	2021
Population in lakhs	y	30	35	41	48	58	70

Estimate the population for the year 1985.

5. (a) Deduce trapezoidal rule for numerical integration. 5

(b) Use the midpoint rule with $M = 5$ to approximate the integral

$$\int_{-1}^{1} \frac{dx}{1+x^2}$$ 5

(c) Evaluate $\int_0^{10} x^2 dx$ by Simpson's $\frac{1}{3}$rd rule. 5

Or

Evaluate $\int_{0\cdot2}^{0\cdot6} \frac{dx}{1+x}$ by Boole's rule correct to three decimal places, use $n = 4$.

6. (a) Find $y(0, 2)$, by Euler's method, from the equation $\frac{dy}{dx} = x^3 + y$, $y(0) = 1$, correct up to four decimal places, taking $h = 0\cdot1$. 4

(b) Write the computational formulae for Runge-Kutta method of order two. 6

Or

Using Runge-Kutta method of fourth order, find the numerical solution at $x = 0\cdot2$ for

$$\frac{dy}{dx} = 1 + y + x^2, \; y(0) = 0 \cdot 5$$

taking $h = 0\cdot2$.

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4 SEM TDC MTMH (CBCS) C 9

2025

(May/June)

MATHEMATICS

(Core)

Paper : C–9

(Riemann Integration and Series of Functions)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Define upper and lower sums with reference to a partition of a closed interval and a function bounded thereat. 2

(b) If f is a bounded function on $[a, b]$ and P, Q are any two partitions of $[a, b]$, then show that $L(f, P) \leq U(f, Q)$. 3

2. Answer any *five* of the following questions : 5×5=25

(a) Let $f:[a, b] \to \mathbb{R}$ be bounded and monotonic. Show that f is integrable on $[a, b]$.

(b) Let $f:[a, b] \to \mathbb{R}$ be continuous. Show that f is integrable on $[a, b]$.

(c) Let $f:[a, b] \to \mathbb{R}$ be integrable. Show that $|f|$ is integrable on $[a, b]$ and

$$\left|\int_a^b f dx\right| \leq \int_a^b |f|\, dx$$

(d) Let $f:[a, b] \to \mathbb{R}$ be integrable. Then show that f^2 is integrable on $[a, b]$.

(e) Let $f:[a, b] \to \mathbb{R}$ be integrable. Then show that indefinite integral F of f is continuous on $[a, b]$ and differentiable at a point $c \in [a, b]$ provided f is continuous at c.

(f) Let f be continuous on $[a, b]$. Show that $\exists\ c \in [a, b]$ such that

$$\frac{1}{b-a}\int_a^b f dx = f(c)$$

(g) Show that the function $f(x) = x^2$ on $[0, 1]$ is integrable and hence compute

$$\int_0^1 x^2 dx$$

3. (a) Test the convergence of any *one* of the following : 5

(i) $\int_0^1 x^{m-1}(1-x)^{n-1} dx$

(ii) $\int_0^\infty \frac{x^{2m}}{1+x^{2n}} dx$

(b) Show that

$$\beta(l, m) = \frac{(l-1)!\,(m-1)!}{(l+m-1)!};\ l, m \in \mathbb{N}$$ 5

4. Answer any *five* of the following questions : 5×5=25

(a) Define pointwise and uniform convergence of a sequence of functions. Give one example each.

(b) Let $f_n : J \to \mathbb{R}$; $J \subseteq \mathbb{R}$ converge uniformly on J to f. Let each f_n be continuous at $a \in J$. Show that f is continuous at a.

(c) Let $f_n, f : J \to \mathbb{R}$; $J \subseteq \mathbb{R}$ be functions and

$$M_n = \inf\{|f_n(x) - f(x)| : x \in J\}$$

provided it exists. Show that $\{f_n\}$ converges uniformly to f if and only if $M_n \to 0$.

(d) Let $f_n : (a, b) \to \mathbb{R}$ be differentiable and $\{f_n'\}$ converges uniformly to another function g. Let there exist $c \in (a, b)$ such that the real sequence $\{f_n(c)\}$ converges. Show that $\{f_n\}$ converges uniformly to a continuous function $f : (a, b) \to \mathbb{R}$.

(e) Let $f_n : (a, b) \to \mathbb{R}$ be differentiable. Let the sequences $\{f_n\}$ and $\{f_n'\}$ converge uniformly to $f, g : (a, b) \to \mathbb{R}$. Then show that f is differentiable and $f' = g$.

(f) State and prove Cauchy criterion for uniform convergence of a series of function Σf_n.

5. (a) Define a power series with an example. 2

(b) Define radius of convergence of a power series and the interval of convergence. 2

(c) State Weierstrass approximation theorem. 2

(d) State and prove Abel's limit theorem. 4

(e) If a power series $\Sigma a_n x^n$ converges for $|x| < R$ such that $f(x) = \Sigma a_n x^n$; $|x| < R$, show that $\Sigma a_n x^n$ converges uniformly on $[R - \varepsilon, R + \varepsilon]$ for any choice of $\varepsilon > 0$, and f is continuous and differentiable and $f'(x) = \Sigma n a_n x^{n-1}$; $|x| < R$. 5

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4 SEM TDC MTMH (CBCS) C 10

2025

(May/June)

MATHEMATICS

(Core)

Paper : C–10

(Ring Theory and Linear Algebra—I)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Give an example of a ring that has no unity. 1

(b) Show that the ring

$$M_2 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \right\}$$

under matrix addition and multiplication is not an integral domain. 3

(c) Prove that every non-zero element of $\mathbb{Z}_n$ is a unit or a zero divisor. 3

Or

Let p be a prime. Show that $\mathbb{Z}_p$ is a field.

(d) Answer any *two* of the following : 4×2=8

(i) Prove that the characteristic of an integral domain is 0 or a prime.

(ii) Let R be a commutative ring with unity. Let I be an ideal of R. Show that $\frac{R}{I}$ is an integral domain if and only if I is a prime ideal.

(iii) Let R be a ring and S be a non-empty subset of R. If S is closed under subtraction and multiplication, then show that S is a subring of R.

(e) Show that the ideal $\langle x \rangle$ is a prime in $\mathbb{Z}[x]$ but not maximal in $\mathbb{Z}[x]$. 4+1=5

Or

Let R be a commutative ring with unity and I be an ideal of R. Show that I is maximal if and only if $\frac{R}{I}$ is a field. 5

2. (a) Define ring homomorphism. Give an example. 1+1=2

(b) Let R and S be two rings and $f : R \to S$ be a ring homomorphism. Prove that $f(-a) = -f(a),\ \forall a \in R$. 2

(c) Let $f : R \to S$ be a ring homomorphism from ring R to ring S. Show that the set

$$\ker f = \{x \in R : f(x) = 0\}$$

is an ideal of R. 3

(d) Answer any *two* of the following : 4×2=8

(i) Determine all ring homomorphisms from $\mathbb{Z}$ to $\mathbb{Z}$.

(ii) Let R be ring with unity e. Show that the map $f : \mathbb{Z} \to R$ defined by

$$f(n) = ne \quad \forall n \in \mathbb{Z}$$

is a ring homomorphism.

(iii) Let R and S be two rings and $f : R \to S$ be a ring homomorphism. Show that

$$\frac{R}{\ker f} \approx f(R)$$

3. (a) Let L be a line in $\mathbb{R}^2$, and the origin is not on L. Is L a subspace of $\mathbb{R}^2$? 1

(b) Show that the set

$$\left\{\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}2\\1\\3\end{pmatrix}, \begin{pmatrix}4\\2\\6\end{pmatrix}\right\}$$

is not linearly independent in $\mathbb{R}^3$. 2

(c) Let H and K be two subspaces of a vector space V. Show that the sum

$$H + K = \{h + k : h \in H, k \in K\}$$

is a subspace of V. 3

(d) Let

$$A = \begin{pmatrix}1 & 2 & 4\\0 & 1 & 2\\-1 & 3 & 6\end{pmatrix}$$

Find a basis for col A. 3

(e) Let V be a finite-dimensional vector space and H be a subspace of V. Prove that $\dim H \le \dim V$. 6

Or

Let V be a vector space and

$S = \{v_1, \ldots, v_p\} \subseteq V$ and $H = \text{span}\{v_1, \ldots, v_p\}$

Let one vector in S say v_k be a linear combination of the remaining vectors in S. Show that the set $S - \{v_k\}$ spans H.

4. (a) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map such that

$$T\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}, \ \forall \begin{pmatrix}x\\y\end{pmatrix} \in \mathbb{R}^2$$

Find the image of $\begin{pmatrix}1\\-1\end{pmatrix}$ under T. 1

(b) Show that the map $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x\\y\\-z\end{pmatrix}, \ \forall \begin{pmatrix}x\\y\\z\end{pmatrix} \in \mathbb{R}^3$$

is a linear map. 2

(c) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a map defined by

$$T\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x+y\\x-y\end{pmatrix}, \ \forall \begin{pmatrix}x\\y\end{pmatrix} \in \mathbb{R}^2$$

Find ker T and range of T. 1+1=2

(d) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map that rotates each point in $\mathbb{R}^2$ about the origin through an angle θ, with counter-clockwise rotation for a positive angle. Find the standard matrix for T. 3

(e) Answer any *three* of the following : 4×3=12

(i) Let $T: V \to W$ be a linear map from finite-dimensional vector space V to W. Show that $\dim V = \text{rank } T + \text{nullity } T$.

(ii) Consider the bases

$$B = \left\{ \begin{pmatrix} 1 \\ -3 \end{pmatrix}, \begin{pmatrix} -2 \\ 4 \end{pmatrix} \right\} \text{ and } C = \left\{ \begin{pmatrix} -7 \\ 9 \end{pmatrix}, \begin{pmatrix} -5 \\ 7 \end{pmatrix} \right\}$$

for $\mathbb{R}^2$. Find the change of coordinate matrix from B to C.

(iii) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be the linear map defined by

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x+y \\ 5x+7y \\ x+3y \end{pmatrix}, \ \forall \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$$

Find the standard matrix for T.

(iv) Define a basis for a vector space V. Let $B = \{b_1, \ldots, b_n\}$ be a basis for V. Let $x \in V$. Show that $\exists$ unique scalars $c_1, \ldots, c_n$ such that

$$x = c_1 b_1 + \ldots + c_n b_n$$

(f) Answer any *two* of the following : 5×2=10

(i) Let V and W be two vector spaces and $T: V \to W$ be a linear map. Show that T is one-one if and only if $\ker T = \{0\}$.

(ii) Let V and W be vector spaces and let $\dim V = n$, $\dim W = m$. Let B and C be ordered bases for V and W respectively. Let $T: V \to W$ be a linear map. Find the matrix for T relative to B and C.

(iii) Let V and W be vector spaces and $T: V \to W$ be a linear map. If T is invertible, then show that the inverse function $T^{-1}: W \to V$ is a linear map.

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