Total No. of Printed Pages—15

5 SEM TDC DSE MTH (CBCS)
1.1/1.2/1.3 (H)

2024

(November)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–1

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : DSE–1.1

(ANALYTICAL GEOMETRY)

1. Answer the following questions :

(a) Write the vertex of the conic $(y-2)^2 = 2(x+2)$. 1

(b) Write the processes to sketch the ellipse. 4

(c) Find the focus, vertex, equation of directrix and length of the latus rectum of the conic $x = y^2 - 4y + 2$. 4

(d) Describe the graph of the curve $3(x-2)^2 + 4(y+1)^2 = 12$. Also find its centre and foci. 6

Or

Describe the graph of the hyperbola $x^2 - y^2 - 4x + 8y - 21 = 0$ and sketch its graph.

2. Answer the following questions :

(a) Write the equation of the tangent to the parabola $x^2 = 4ay$ at the point (x_1, y_1). 1

(b) Write True or False : 1

An ellipse is the set of all points in the plane that are equidistant from a fixed line and a fixed point not on the line.

(c) Suppose that an ellipse has semi-major axis a, semi-minor axis b and foci $(\pm c, 0)$. Then write the expression c in terms of a and b. 1

(d) Find the equation of the parabola that has its vertex at $(1, 2)$ and focus at $(4, 2)$. Also state the reflection property of the parabola. 6

(e) Find the equation of the ellipse with foci $(0, \pm 2)$ and major axis with end-points $(0, \pm 4)$ and also sketch it. 6

Or

Find the equation and sketch the curve of the hyperbola whose foci $(-3, -3)$ and $(3, 3)$.

3. Answer the following questions :

(a) Write the condition that the quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

represents ellipse. 1

(b) Find the new coordinates of the point $(2, 4)$ if the coordinate axes are rotated through an angle $\theta = \pi/6$. 2

(c) Consider the equation $x^2 - xy + y^2 - 6 = 0$. Rotate the coordinate axes to remove the xy-term. Then identify the type of conic represented by the equation and sketch its graph. 6

(d) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 30°$.

(i) Find the $x'y'$-coordinate of the point whose xy-coordinates are $(2, 4)$.

(ii) Find an equation of the curve

$$2x^2 + 2\sqrt{3}xy = 3$$

in $x'y'$-coordinates. 6

Or

Identify and sketch the curve

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$$

4. Answer the following questions :

(a) Write the equation of the sphere whose end-points of the diameter is given. 1

(b) Write the standard equation of hyperbola of one sheet. 1

(c) Write True or False : 1

Curve of intersection of two spheres is a sphere.

(d) Write the centre and radius of the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$ 2

(e) Find the equation of the sphere whose centre is (2, –3, 4) and radius is 6 units. 5

(f) Find the equation of the sphere which passes through the points (1, –3, 4), (1, –5, 2), (1, –3, 0) and whose centre lies on the line $x + y + z = 0$. 5

Or

Find the equation of the sphere of centre at (1, 2, 3) and touching a plane at (2, 1, 3).

5. Answer the following questions :

(a) Find the radius and centre of the circle

$$x^2 + y^2 + z^2 - x - y - z - 1 = 0, \quad x + y + z = 0$$ 5

(b) Find the equation of the sphere for which the circle

$$x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, \quad x + y + z = 3$$

is a great circle. 5

Or

Find the equations of the spheres which pass through the

$$x^2 + y^2 + z^2 - 2x + 2y + 4z - 3 = 0; \quad 2x + y + z - 4 = 0$$

and touch the plane $3x + 4y - 14 = 0$.

6. Answer the following questions :

(a) Show that the plane $2x + y - z = 12$ touches the sphere $x^2 + y^2 + z^2 = 24$ and find its point of contact. 5

(b) Classify and sketch the surface $9x^2 + 4y^2 + z^2 = 36$. 5

Or

Classify and sketch the surface $z = x^2 + 4y^2$.

Paper : DSE–1.2

(PORTFOLIO OPTIMIZATION)

1. Answer any *six* of the following questions : 1×6=6

(a) What is mutual fund?

(b) Define diversification.

(c) Write one advantage of Sharpe performance model.

(d) What is market timing?

(e) What is beta of a portfolio?

(f) Define risk-free asset.

(g) What is the value of correlation between risky asset and risk-free asset?

2. Answer any *six* of the following questions : 4×6=24

(a) Write a short note on investment objective and investment constraints. 4

(b) What are the different components of systematic risk? 4

(c) How would you differentiate risk from uncertainty? 4

(d) What is the difference between historical and expected returns? Write two measures of mean historical returns. 3+1=4

(e) If an investment that costs ₹ 400 and is worth ₹ 500 after being held for two years, find annual holding period return (annual HPR) and annual holding period yield (annual HPY). 2+2=4

(f) Discuss four different types of risk of an investment. 4

(g) Calculate the expected rate of return and the risk in terms of variance of the following economic scenarios : 2+2=4

Economic Conditions	*Probability*	*Rate of Return*
Strong economy	0·25	0·20
Weak economy	0·25	−0·20
No major change in economy	0·50	0·10

3. (a) How can risk of an asset be calculated? 2

(b) State one-fund theorem. 2

(c) What do you mean by efficient frontier? 2

4. Answer any *two* of the following questions : 6×2=12

(a) Write the assumptions of capital market theory. Explain briefly.

(b) Discuss some of the disadvantages of Markowitz model.

(c) In what way two-factor model is better than one-factor model? Justify.

5. Describe variance and standard variation of returns for a portfolio of investments. 7

Or

Find the covariance of rates of returns of US stocks and US bonds as given below :

2020	*US Stock Index* (R_i)	*US Bond Index* (R_j)
January	−3·60	1·58
February	3·10	0·40
March	6·03	−0·85
April	1·58	1·05
May	−7·99	1·71
June	−5·24	1·87
July	7·01	0·68
August	−4·51	2·01
September	8·92	0·02
October	3·81	−0·16
November	0·01	0·70
December	6·68	−1·80

If standard deviations of both scenarios are $\sigma_i = 5.56$ and $\sigma_j = 1.22$, then find the correlation. 4+3=7

6. Write the formula for beta of a portfolio. Interpret beta of 1·20 and 0·70. 1+2=3

7. Answer any *three* of the following questions : 5×3=15

   (a) Distinguish between capital market line (CML) and security market line (SML).

   (b) State the limitations of Jensen's performance index model.

   (c) Discuss the assumptions of capital asset pricing model (CAPM).

   (d) Discuss the effects of combining securities in portfolio.

8. Describe Treynor portfolio performance measure with example. 7

Or

Consider the following information on three mutual funds *P*, *Q* and *R*, and the market index :

	Mean Return	*SD*	*Beta*
P	15%	20%	0·90
Q	17%	24%	1·10
R	19%	27%	1·20
Market Index	16%	20%	1·00

The mean risk-free rate is 10%. Calculate the Treynor's measure and Sharpe measure for the three mutual funds. 4+3=7

Paper : DSE–1.3

(FINANCIAL MATHEMATICS)

1. (a) Write the inverse supply function of the function $30q+5p=80$. 1

(b) Write the revenue, if p is the price and q is the number of quantities sold. 1

(c) For demand and supply functions, the equilibrium set will always a singleton set. State true or false. 1

(d) After introduction of excise tax, among the supply and demand functions, write which function remains same. 1

(e) Write two reasons for introducing excise tax. 2

(f) Find the solution of the recurrence equation $5y_t = 3y_{t-1} + 6$, given $y_0 = 4$. 4

Or

Show that the present value of an annuity I for N years, given the fixed interest rate r is

$$P = \frac{I}{1+r} + \frac{I}{(1+r)^2} + \frac{I}{(1+r)^3} + \cdots + \frac{I}{(1+r)^N}$$

2. Answer any *two* of the following : 4×2=8

(a) Describe cobweb model.

(b) Describe general linear case.

(c) Determine whether cobweb model predicts stable or unstable equilibrium for the market with supply and demand functions $2p-3q=12$ and $2p+q=20$ respectively.

3. (a) Define fixed cost. 1

(b) Write one difference between maximum point and a point of inflection. 1

(c) Find the critical points of the function $f(x) = x^3 - 12x^2 + 21x + 12$. 2

(d) Find the extreme values of the function $f(x) = x^4 - 8x^3 + 16x^2 - 7$ in the interval $[1, 4]$. 4

4. (a) Write when demand is elastic. 1

(b) If $C(q) = 100 + 20q - 2q^2 + 6q^3$ be the cost function, then find the fixed cost. 2

(c) Define break-even point and variable cost. 2+2=4

(d) Show that at the startup point, marginal cost is equal to average variable cost. 5

Or

Find the elasticity of demand for the demand function

$$D = \{(q, p) : q(1+p^2) = 50\}$$

Also find the values of p when the demand is elastic and inelastic.

5. (a) Let

$$f(x, y) = x^3y^2 + x^2y$$

Find

$$\frac{\partial f}{\partial x}$$ 1

(b) Let p_1 and p_2 are the selling prices of two items X and Y. Write the revenue of producing q_1 units of X and q_2 units of Y. 1

(c) Find the critical point(s) of the profit function

$$I(x, y) = 5x + 24y - 1\cdot5x^2 - 2y^2 + xy - 5$$ 3

(d) Classify the critical points of the function $f(x, y) = y^3 + 3xy - x^3$ and find the extreme values. 5

Or

A firm produces two goods A and B, with demand functions $x = 12 - p^A$, $y = 18 - p^B$. Firm's cost function is $c(x, y) = x^2 + y^2 + 2xy$. Find the maximum achievable profit.

6. (a) Define portfolio. 1

(b) Write when a portfolio is called an arbitrage portfolio. 1

(c) Answer any *two* questions from the following : 5×2=10

(i) Describe technology matrix.

(ii) Let

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } A^n = \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix}$$

$n \geq 2$. Find recurrence equations for a_n, b_n, c_n, d_n.

(iii) For input-output model with two industries, the coefficient matrix is

$$A = \begin{bmatrix} 0\cdot3 & 0\cdot1 \\ 0\cdot2 & 0\cdot4 \end{bmatrix}$$

Determine the production schedule x in terms of external demand d.

7. (a) Cash flow may be negative. State true or false. 1

(b) Write any one form of hedging. 1

(c) Interest may be called by an another name. Write that name. 1

(d) Write the meaning of the cash flow stream (–100, 200). 1

(e) Write the risk aversion principle. 2

8. (a) Write under what type of interest, capital exhibits geometric growth. 1

(b) If r is the 1-year interest rate, then write 1-year discount factor. 1

(c) Define effective interest rate. 2

(d) Find the internal rate of return of the cash flow (–1, 1, 0, 1). 5

Or

Describe Macaulay duration.

(e) Explain callable bonds. 2

(f) Find the future value of the cash flow stream (–20, 10, 10, 10) when the periods are years and the interest rate is 10%. 3

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5 SEM TDC DSE MTH (CBCS)
2.1/2.2/2.3/2.4 (H)

2024

(November)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–2.1/2.2/2.3/2.4

The figures in the margin indicate full marks for the questions

Paper : DSE–2.1

(MATHEMATICAL MODELLING)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

1. *(a)* Find the value of $\Gamma\left(\frac{5}{2}\right)$. 1

(b) Is the point $x=0$ an ordinary point of the equation

$$y'' + x^2 y' + x^{1/2} y = 0\,?$$ 1

(c) Find the value of $L\{\cosh kx\}$. 2

(d) Find the inverse Laplace transform of

$$F(s) = \frac{1}{s^2(s-a)}$$ 3

2. (a) Investigate the nature of the point $t=0$ for the differential equation

$$t^4 y'' + (1-\cos t)y + (t^2 \sin t)y' = 0$$ 2

(b) Find a power series solution of the differential equation $(x-3)y' + 2y = 0$. Determine the radius of convergence of the resulting series. 4+1=5

3. (a) Use Laplace transforms to solve the initial value problem

$$x'' - x' - 6x = 0;\ x(0) = 2,\ x'(0) = -1$$ 5

Or

$$x'' + 8x' + 15x = 0;\ x(0) = 2,\ x'(0) = -3$$

(b) Find the Frobenius series solutions of

$$xy'' + 2y' + xy = 0$$ 6

4. (a) Define 'goal programs'. 1

(b) Use the linear congruence method to generate 10 random numbers using $a=5,\ b=1$ and $c=8$. 2

(c) Why is sensitivity analysis important in linear programming? 3

5. (a) Define feasible solution. 1

(b) Solve the following model algebraically : 4

Maximize $25x + 30y$

subject to

$$20x + 30y \le 690$$
$$5x + 4y \le 120$$
$$x,\ y \ge 0$$

(c) Explain middle-square method and use it to generate random numbers taking $x_0 = 2041$. Does this method have any drawbacks? Illustrate. 2+3+1=6

6. Answer any *three* of the following questions : 6×3=18

(a) Using Monte Carlo simulation, write an algorithm to calculate the volume of the sphere $x^2 + y^2 + z^2 \le 1$ that lies in the first octant, $x > 0,\ y > 0,\ z > 0$.

(b) Use the golden-section search method with a tolerance of $t = 0{\cdot}2$ to minimize

$$f(x) = x^2 + 2x, \; -3 \le x \le 6$$

(c) Use the curve-fitting criterion to minimize the sum of the absolute deviations for the model $y = ax^3$ and data set

x	:	7	14	21	28	35	42
y	:	8	41	133	250	280	297

(d) Using simplex method, solve the following problem :

Maximize $3x + y$

subject to

$$2x + y \le 6$$
$$x + 3y \le 9$$
$$x, y \ge 0$$

(e) Fit the model $y = cx$ to the following data using Chebyshev's criterion to minimize the largest deviation :

x	:	1	2	3
y	:	2	5	8

Paper : DSE–2.2

(MECHANICS)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

UNIT—I

1. *(a)* Define couple moment. A force

$$\vec{F} = (10\hat{i} + 6\hat{j} + 3\hat{k})$$

acts at position (3, 0, 2). At point (0, 2, –3), an equal but opposite force $-\vec{F}$ acts. Calculate the couple moment. 1+2=3

(b) Show that any number of coplanar couples acting on a body is equivalent to a single couple whose moment is equal to the algebraic sum of moments of the couples. 6

Or

Derive the general equations of equilibrium.

(c) An electric light fixture weighing 15 N hangs from a point C by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal. Draw the free-body diagram and determine forces in the strings AC and BC. 5

(d) A parallel system of forces is such that a 20-N force acts at position $x = 10$ m, $y = 3$ m; a 30-N force acts at position $x = 5$ m, $y = -3$ m; a 50-N force acts at position $x = -2$ m, $y = 5$ m.

(i) If all forces act along negative z-direction, give the simplest resultant force and its line of action.

(ii) If the 50-N force acts along positive z-direction and the others act along negative direction, find the resultant. 4+2=6

UNIT—II

2. (a) What do you mean by coefficient of friction? Write down the dimension of coefficient of friction. 2+2=4

(b) The horizontal position of the 500 kg rectangular block of concrete is adjusted by the 5° wedge under the action of the force *P*. If the coefficient of friction for both wedge surfaces is 0·30 and the coefficient of friction between horizontal surface and block is 0·60, then determine the least force *P* required to move the block. 6

Or

A strongbox of mass 75 kg rests on a floor. The static coefficient of friction for the contact surface is 0·20. Calculate the largest force *P* and highest position *h* for applying this force that will not allow the strongbox to either slip on the floor or to tip.

(c) Find the centroid of the area under the half-sine wave. 5

(d) Show that

$$I_{xx} = \frac{bh^3}{12},\ I_{yy} = \frac{b^3h}{12} \text{ and } I_{xy} = \frac{b^2h^2}{24}$$

for the right-angled triangle of base *b* and height *h*. 5

(e) State and prove the theorem of Pappus-Guldinus. 5

UNIT—III

3. (a) Write down the statement of law of conservation of mechanical energy. 2

(b) Define constant force field. 1

(c) Given the following conservative force field

$$\vec{F} = (10z + y)\hat{i} + (15yz + x)\hat{j} + \left(10x + \frac{15y^2}{2}\right)\hat{k}$$

Find the force potential. Also, calculate the work done by $\vec{F}$ on a particle going from

$$\vec{r_1} = 10\hat{i} + 2\hat{j} + 3\hat{k} \text{ to } \vec{r_2} = -2\hat{i} + 4\hat{j} - 3\hat{k}$$

4+3=7

Or

Show that the moment of the resultant force on a particle about a point, fixed in an inertial reference, is equal to the time rate of change of moment of the linear momentum of the particle relative to the inertial reference frame. 7

4. (a) State and prove Chasles' theorem. 7

(b) A cylinder of radius R rotates about its own axis with an angular speed ω. If the total mass is M, show that the kinetic energy is

$$\frac{1}{4}MR^2\omega^2$$ 6

(c) Establish the relation between acceleration vectors of a particle for two systems of references moving arbitrarily relative to each other. 6

(d) Derive the moment of momentum equation for a single particle. 6

Paper : DSE–2.3

(NUMBER THEORY)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. (a) Exhibit a complete residue system modulo 7 consisting of multiples of 3. 2

(b) If $\tau(n) = 2$, then what conclusion can be drawn for the number n? 1

(c) Write the values of $\mu(49)$ and $\left[-\frac{1}{3}\right]$. 2

(d) How many primitive roots of 9 are there? 1

(e) Find the value of the Legendre symbol

$$\left(\frac{2}{17}\right)$$ 1

(f) What do you understand by primitive root of an integer n? 1

2. (a) Find the solutions in positive integers for any *one* of the following : 5

(i) $18x + 5y = 48$

(ii) $15x + 7y = 111$

(b) Prove that $a^{21} \equiv a \pmod{15}$ for all a. 3

3. Answer any *two* of the following : 5×2=10

(a) Arrange the integers $2, 3, 4, \cdots, 21$ in pairs a and b that satisfy

$$ab \equiv 1 \pmod{23}$$

(b) Solve the following simultaneous congruences :

$$x \equiv 5 \pmod{7}$$
$$x \equiv 7 \pmod{11}$$
$$x \equiv 3 \pmod{13}$$

(Continued)

(c) State and prove Wilson's theorem. Is the converse of Wilson's theorem true?

4. (a) What is meant by number theoretic function? When is it said to be a multiplicative function? 1+1=2

(b) Find the value of $\tau(900)$. 3

(c) Prove that the Mobius μ function is a multiplicative function. 3

(d) Prove that for a prime p

$$\phi(p^k) = p^k\left(1 - \frac{1}{p}\right)$$

Hence find $\phi(32)$. 4

5. Answer any *three* of the following : 5×3=15

(a) For each positive integer $n \geq 1$, prove that

$$n = \sum_{d|n} \phi(d)$$

(b) State and prove Mobius inversion formula.

(Turn Over)

(c) For $n > 1$, prove that the sum of the positive integers less than n and relatively prime to n is

$$\frac{1}{2} n\phi(n)$$

(d) If $\gcd(m, n) = 1$, then prove that the set of positive divisors of mn consists of all products $d_1 d_2$ where $d_1 | m$, $d_2 | n$ with $\gcd(d_1, d_2) = 1$.

6. (a) Write the conditions for which an integer n fails to have primitive root. 2

(b) Define the words 'plaintext' and 'ciphertext'. 2

(c) Determine whether 2 is quadratic residue or non-residue of 13. 3

(d) Find a Pythagorean triple of the form 16, y, z. 2

(e) Let p be an odd prime and $(a, p) = 1$. Prove that 3

$$\left(\frac{a}{p}\right) \equiv a^{(p-1)/2} \pmod p$$

7. Answer any *three* of the following : 5×3=15

(a) Solve :

$$x^2 + 7x + 10 \equiv 0 \pmod{11}$$

(b) If order of a modulo $p = 3$, where p is prime, then show that order of $(a+1)$ modulo p is 6.

(c) If p is an odd prime, then there are precisely $\frac{(p-1)}{2}$ quadratic residues and $\frac{(p-1)}{2}$ quadratic non-residues of p.

Prove it.

(d) Solve :

$$x^2 \equiv 7 \pmod{3^3}$$

(e) Prove that there are infinitely many primes of the form $8k - 1$.

Paper : DSE–2.4

(**BIOMATHEMATICS**)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

UNIT—I

1. Answer any *two* of the following questions : $7\frac{1}{2}\times2=15$

(a) A population is originally 100 individuals, but because of the combined effects of births and deaths, it triples each hour.

(i) Make a table of population size for $t=0$ to 5, where t is measured in hours.

(ii) Give two equations modeling the population growth by first expressing P_{t+1} in terms P_t and then expressing ΔP in terms of P_t.

(iii) What can you say about the birth and death rates for this population?

(b) In the early stages of the development of a frog embryo, cell division occurs at a fairly regular rate. Suppose you observe that all cells divide, and hence

the number of cells double, roughly every half-hour.

(i) Write down an equation modeling this situation. You should specify how much real-world time is required by an increment of 1 in t and what the initial number of cells is.

(ii) Produce a table and graph of the number of cells as a function of t.

(c) Obtain a simple prey-predator model explaining in detail the assumptions taken. Also find the equilibrium positions.

UNIT—II

2. Answer any *two* of the following questions : $7\frac{1}{2}\times2=15$

(a) Consider the SI epidemic model. If the contact rate is 0·001 and the number of susceptibles is 2000 initially, determine—

(i) the number of susceptibles left after 3 weeks;

(ii) the density of susceptible when the rate of appearance of new cases is a maximum;

(iii) the time (in week) at which the rate of appearance of new cases is a maximum;

(iv) the maximum rate of appearance of new cases.

(b) In an SIS model, if the infection is spread only by a constant number of carriers, then show that

$$I(t) = \left(I_0 - \frac{\alpha CN}{\alpha C + \beta}\right) e^{[-(\alpha C+\beta)t]} + \frac{\alpha CN}{\alpha C + \beta}$$

where I and C are the number of infectives and carriers; N is total population; α and β are contact rate and susceptible rate respectively; I_0 is the infectives at $t = 0$.

(c) Let x and y respectively denote the proportion of susceptibles and carriers in a population. Suppose the carriers are indentified and removed from the population at a rate β, so that

$$\frac{dy}{dt} = \beta y$$

Suppose also that the disease spreads at a rate proportional to the product of x and y, thus

$$\frac{dx}{dt} = -\alpha xy$$

(i) Determine the proportions of carriers at any time t, where $y(0) = y_0$.

(ii) Use (i) above to find the susceptibles at time t, where $x(0) = x_0$.

(iii) Find the proportion of population that escapes the epidemic.

UNIT—III

3. Answer any *two* of the following questions : 7½×2=15

(a) Consider the competition models for two species with populations N_1 and N_2

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1}{K_1} - b_{12}\frac{N_2}{K_1}\right)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - b_{21}\frac{N_1}{K_2}\right)$$

where only one species N_1 has limited carrying capacity. Investigate their stability and sketch the phase-plane trajectories. [Here, K_1, K_2 are carrying capacities; r_1, r_2 are linear birth rates of the populations N_1 and N_2 respectively; b_{12}, b_{21} measure the competitive effects of N_2 on N_1 and N_1 on N_2 respectively.] 4+3½=7½

(b) What is Routh-Hurwitz criterion? Explain with reference to multiple species communities. 2+5½=7½

(c) Discuss bifurcation and limit cycle with respect to any biological model. 7½

UNIT—IV

4. Answer any *two* of the following questions : 7½×2=15

(a) Write a short note on any *one* of the following :

(i) One-species model with diffusion

(ii) Two-species model with diffusion

(b) For a blood vessel of constant radius R, length L and driving force $P = p_1 - p_2$, show that the average velocity of the flow is equal to half of the maximum velocity and the resistance is proportional to $\frac{L}{R^4}$.

(c) Consider arterial blood viscosity $\mu = 0{\cdot}027$ poise.

If the length of the artery is 2 cm, radius 8×10^{-3} cm and $P = p_1 - p_2 = 4 \times 10^3$ dynes/cm^2, then find—

(i) $q_z(r)$ and the maximum peak velocity of blood;

(Continued)

(ii) the shear stress at the wall.

(Here q_z denotes velocity along z-axis, p_1 and p_2 denote pressure at two ends of the artery.)

UNIT—V

5. Answer any *two* of the following questions : 10×2=20

(a) Let D & d and W & w respectively denote allele for tall & dwarf and round & wrinkled seeds of peas. Find the outcome of the product $DdWw \times ddWw$ using Punneet square or using probability. Also find the probability that the progeny of $DdWw \times ddWw$ is dwarf with round seeds. 6+4=10

(b) Explain in detail the Hardy-Weinberg equilibrium, mentioning the assumptions considered for the equilibrium.

(c) Compare and contrast stage structure model with age structure model.

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