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6 SEM TDC DSE MTH (CBCS) 2 (H)

2024

(May)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–2

(Linear Programming)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Define slack and surplus variables in a linear programming problem. 2

(b) Solve by simplex method

$$\text{Min } Z = x_1 - 3x_2 + 2x_3$$

subject to

$$3x_1 - x_2 + 3x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$\text{and} \quad x_1, x_2, x_3 \ge 0$$ 7

Or

Explain the various steps of the simplex method involved in the computation of an optimum solution to a linear programming problem. 7

(c) Answer any *two* : 8×2=16

(i) Solve the following linear programming problem by two-phase simplex method :

Min $Z = x_1 + x_2$

subject to

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$

and $x_1, x_2 \geq 0$

(ii) Use Big-M method to solve the following Linear Programming Problem :

Max $Z = x_1 + 2x_2 + 3x_3 - x_4$

subject to

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$

and $x_1, x_2, x_3, x_4 \geq 0$

(iii) Solve the Linear Programming Problem :

Min $Z = 5x_1 + 3x_2$

subject to

$$2x_1 + 4x_2 \leq 12$$
$$2x_1 + 2x_2 = 10$$
$$5x_1 + 2x_2 \geq 10$$

where $x_1, x_2 \geq 0$

by Big-M method.

2. (a) If the objective of the primal is to maximize, then write the objective of the dual. 1

(b) Write the dual of the following linear programming problem :

Max $Z = x_1 - x_2 + 3x_3$

subject to

$$x_1 + x_2 + x_3 \leq 10$$
$$2x_1 - x_2 - x_3 \leq 2$$
$$2x_1 - 2x_2 - 3x_3 \leq 6$$

and $x_1, x_2, x_3 \geq 0$ 4

(c) Answer any *two* from the following : 5×2=10

(i) Write the mathematical formulation of the dual linear programming problem in symmetrical form.

(ii) Prove that the dual of a dual is primal.

(iii) Give an economic interpretation of dual variables.

3\. *(a)* Write the necessary and sufficient condition for a feasible solution to a transportation problem. 2

(b) Write the conditions for a non-degenerate basic feasible solution. 2

(c) Answer any *two* : 8×2=16

(i) Describe the computational procedure of the MODI method in a transportation problem.

(ii) Find the initial basic feasible solution using Vogel's Approximation method and find the optimal solution :

	D_1	D_2	D_3	D_4	*Supply*
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

(iii) A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effectiveness matrix :

		Employees				
		I	*II*	*III*	*IV*	*V*
Jobs	*A*	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

How should the jobs be allocated, one per employee, so as to minimize the total man hours?

4\. *(a)* What is a strictly determine game in game theory? 1

(b) Answer any *two* : 5×2=10

(i) Solve the following game stating the optimal strategies and the saddle point : 5

$$\begin{bmatrix} 2 & 3 & 2 & 4 & 6 \\ 0 & -2 & 1 & 2 & 1 \\ -1 & 3 & 0 & -1 & 3 \\ 4 & 5 & -1 & 2 & 1 \\ 3 & 2 & -2 & 1 & -2 \end{bmatrix}$$

(ii) Find the value of the 2×2 game algebraically by using mixed strategies : 5

Player A

	B_1	B_2
Player B A_1	2	3
A_2	4	−1

(iii) Solve the following 2×4 game geometrically : 5

Player B

	B_1	B_2	B_3	B_4
Player A A_1	3	2	−1	4
A_2	2	5	6	−2

(c) Solve the game problem by using LP method : 9

Player B

	B_1	B_2	B_3
Player A A_1	1	0	−2
A_2	0	3	2

Or

State the modified dominance property. Reduce the following game to 2×2 game by using dominance and modified dominance property and then solve the game :

Player B

	B_1	B_2	B_3	B_4
A_1	1	2	−2	2
A_2	3	1	2	3
Player A A_3	−1	3	2	1
A_4	−2	2	0	−3

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6 SEM TDC DSE MTH (CBCS) 6 (H)

2024

(May)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–6

(Mathematical Methods)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Let $f(x)$ be a periodic function of period 2π. Write the value of $f(x)$ to which it converges at the end points $x = \pm\pi$. 1

(b) Define Fourier series of a function $f(x)$ in the interval $(a, a+2\pi)$, $a \in R^+$. 2

(c) Find a Fourier series for the function $f(x) = x + x^2$ in the interval $(-\pi, \pi)$. 7

Or

Find the half range cosine and sine series of the function $f(x) = x$ in the interval $(0, \pi)$.

2. (a) Write the value of (i) $L\{2\}$ and (ii) $L\{\sin x\}$. 1+1=2

(b) Find (i) $L\{1-2\sin^2 x\}$ and (ii) $L\{t^2\}$. 2+2=4

(c) State and prove the second shifting theorem of Laplace transform. 4

Or

Find $L\{e^{-t}(2\sinh t + 7\cos 3t)\}$.

(d) Find $L\{t\cos 2t\}$. 4

Or

Find $L\{e^t \cos^2 2t\}$.

(e) Find the following (any *two*) : 3×2=6

(i) $L\{a + bt^2 + c\sqrt{t}\}$

(ii) $L\{(\cos x + 1)^2\}$

(iii) $L\{te^t \sin t\}$

3. (a) Write the value of $L^{-1}\left\{\frac{1}{s-2}\right\}$. 1

(b) Write the value of $L^{-1}\left\{\frac{1}{s^6}\right\}$. 1

(c) Define null function. 1

(d) Find the following (any *one*) : 3

(i) $L^{-1}\left\{\frac{3}{(s-3)^2+3^2}\right\}$

(ii) $L^{-1}\left\{\frac{1}{s^2-6s+10}\right\}$

(e) Find the following (any *one*) : 4

(i) $L^{-1}\left\{\frac{1}{(s^2+4)(s+1)^2}\right\}$

(ii) $L^{-1}\left\{\log\frac{s+7}{s+2}\right\}$

4. (a) Write the Fourier sine integral formula. 1

(b) Write the Dirichlet's conditions for Fourier transform. 2

(c) Find the Fourier sine transform of $f(x) = x$. 3

(d) State and prove the change of scale property of Fourier transform. 5

Or

Prove that

$$F\{x^n f(x)\} = (-i)^n \frac{d^n}{dp^n}[\bar{f}(p)]$$

(e) Answer the following (any *two*) : 7×2=14

(i) Find the Fourier transform of $f(x)$ defined by

$$f(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0, & x>1\end{cases}$$

(ii) Find the inverse Fourier transform $f(x)$ of $F(p)=e^{-|p|y}$.

(iii) Find the Fourier cosine and sine transforms of e^{-ax}, $a>0$.

5. *(a)* Write the value of $L\left\{\frac{\partial y}{\partial x}\right\}$. 1

(b) Find $L\left\{\frac{\partial y}{\partial t}\right\}$. 2

(c) Solve using Laplace transform (any *two*) : 6×2=12

(i) $(D^2+4D+5)y=5$, $y(0)$, $y'(0)=0$, $D\equiv\frac{d}{dt}$

(ii) $(D^2+3D+2)y=e^{-t}$, $y(0)=0$, $y'(0)=1$

(iii) $(D^2+9)y=\sin t$, $y(0)=1$, $y\left(\frac{\pi}{2}\right)=1$

(iv) $\frac{\partial y}{\partial x}=2\frac{\partial y}{\partial t}+y$, $y(x, 0)=6e^{-3x}$

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