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**5 SEM TDC DSE MTH (CBCS)**  
**2.1/2.2/2.3/2.4 (H)**

**2025**

( Nov/Dec )

**MATHEMATICS**

( Discipline Specific Elective )

( For Honours )

Paper : DSE-2.1/2.2/2.3/2.4

*The figures in the margin indicate full marks  
for the questions*

Paper : DSE-2.1

**( MATHEMATICAL MODELLING )**

Full Marks : 60

Pass Marks : 24

Time : 3 hours

1. Answer the following as directed : 1×5=5
- (a) Define ordinary point.
- (b)  $\Gamma(1) = \underline{\hspace{2cm}}$  (Fill in the blank)
- (c) Define Bessel's equation.

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- (d) Find  $L\{1\}$ .  
(e) State a sufficient condition for the existence of Laplace transform of a function.

2. Answer any one of the following :

- (a) Determine whether  $x=0$  is an ordinary point or a regular singular point of the differential equation

$$2x^2 \frac{d^2y}{dx^2} + 7x(x+1) \frac{dy}{dx} - 3y = 0$$

- (b) Find  $L^{-1}\left(\frac{1}{s}\right)$ .

3. Express  $4x^2 - 3x + 2$  in terms of Legendre polynomial.

4. Find Laplace transform (any two) :

(a)  $(t^2 + 1)^2$

(b)  $\cosh at$

(c)  $\frac{1}{\sqrt{\pi t}}$

5. Find the solution in series of

$$(x^2 + 1)y'' + xy' - xy = 0$$

about  $x=0$ .

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6. Show that

$$L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\left(\frac{\pi}{s}\right)} e^{-\frac{1}{4}s}$$

Or

Show that the Laplace transform of the function  $F(t) = t^n$ ,  $-1 < n < 0$  exists, although it is not a function of class A.

7. Answer the following :

(a) What is feasible region?

(b) When is an optimization problem said to be a 'linear program'?

(c) Define dynamic program and stochastic program.

(d) Write the Monte Carlo inventory algorithm.

(e) "Monte Carlo simulation is a deterministic process." Write true or false.

Using Monte Carlo simulation, write an algorithm to calculate the area under the curve  $f(x) = \sqrt{x}$ , over the interval

$$\frac{1}{2} \leq x \leq \frac{3}{2}$$

1+3=4

8. Answer any two of the following :  $5 \times 2 = 10$

(a) A carpenter realizes a net unit profit of ₹ 25 per table and ₹ 30 per bookcase. He has up to 690 board-feet of lumber and up to 120 hours of labour to devote weekly to the project. He estimates that it requires 20 board-feet of lumber and 5 hours of labour to complete a table, and 30 board-feet of lumber and 4 hours of labour to complete a bookcase. Determine how many tables and how many bookcases he should make each week to maximize his profit.

(b) Solve the following linear program algebraically :

$$\begin{aligned} & \text{Maximize } 10x + 35y \\ & \text{subject to } 8x + 6y \leq 48 \\ & \quad 4x + y \leq 20 \\ & \quad y \geq 5 \\ & \quad x, y \geq 0 \end{aligned}$$

(c) What is dichotomous search method? Write dichotomous search algorithm to maximize a function  $f(x)$  over the interval  $[a, b]$ .

9. Answer any two of the following :  $6 \times 2 = 12$

(a) Explain linear congruence and use it to generate 10 random numbers using  $a = 1$ ,  $b = 7$  and  $c = 10$ .

(b) Solve using simplex method :

$$\begin{aligned} & \text{Minimize } 6x + 5y \\ & \text{subject to } x + y \geq 6 \\ & \quad 2x + y \geq 9 \\ & \quad x, y \geq 0 \end{aligned}$$

(c) Use dichotomous search method to maximize  $f(x) = -x^2 - 2x$  over the interval  $-3 \leq x \leq 6$  with a tolerance of  $t = 0.2$  and  $\epsilon = 0.01$ .

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Paper : DSE-2.2

( MECHANICS )

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. (a) Determine the moment about the origin  $O$  of the force

$$\vec{F} = -7N\hat{i} - 2N\hat{j} + 3N\hat{k}$$

which acts at a point  $A$ . The position vectors of  $A$  are (i)  $\vec{r} = 4m\hat{i} + 2m\hat{j} - m\hat{k}$  and (ii)  $\vec{r} = -8m\hat{i} + 3m\hat{j} + 4m\hat{k}$ .

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- (b) What do you mean by distributed force system? Give an example of it. 2+1=3

- (c) Show that two couples in the same plane whose moments are equal and of the same sign are equivalent to one another.

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- (d) A 700 N circus performer causes a 0.15 m sag in the middle of 12 m tight rope with a 5000 N initial tension. What additional tension is induced in the cable? What is the cable tension when the performer is 3 m from the end and the sag is 0.12 m?

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Or

Show that the necessary condition for a rigid body to be in equilibrium is that the resultant force and resultant couple point for any point to be zero vectors.

2. (a) Write down the Coulomb's laws of friction. 2

- (b) The block of weight  $W$  is to be moved up an inclined plane. A rod of length  $C$  with negligible weight is attached to the block and the force  $\vec{F}$  is applied to the top of the rod. If the coefficient of static friction is  $\mu_s$ , determine the maximum length  $C$  for which the block will begin to slide rather than tip in terms of  $d$  (block's breadth) and  $\mu_s$ . 5

- (c) Find  $I_{xx}$ ,  $I_{yy}$  and  $I_{xy}$  for the area bounded by  $y = e^x$  and  $y = -e^x$ . 6

Or

Find the first moments of the area bounded by  $y^2 = 25x$  and  $x$ -axis and  $y = 10$  about  $x$ - and  $y$ -axes.

- (d) Establish the relation between second moments and product of inertia. 5
- (e) Define centroid. Find the centroid of the area bounded by  $y^2 = 4x$  and  $x^2 = 4y$ .  
2+5=7
3. (a) What do you mean by conservative force field? Show that in a conservative force field,  $\vec{F} = -\nabla V$ , where the symbols have their usual meanings. 2+3=5
- (b) For a given conservative force field  

$$\vec{F} = (5z \sin x + y)\hat{i} + (4yz + x)\hat{j} + (2y^2 - 5 \cos x)\hat{k}$$
 find the force potential. What is the work done on a particle starting at the origin and moving in a circular path of radius 2 to form a semicircle along the positive  $x$ -axis? 6
- (c) Derive the moment of momentum equation for a system of particle. 6
- (d) Establish the relation between acceleration vectors of a particle for two systems of references moving arbitrarily relative to each other. 6

Or

Find the kinetic energy of a rigid body rotating about a fixed point.

- (e) Show that the sum of the potential energy and the kinetic energy for a particle remains constant for all times during the motion of a particle.

A particle is dropped with zero initial velocity down a frictionless chub. What is the magnitude of the velocity if the vertical drop during the motion is  $h$  ft? 6

- (f) State and prove Chasles' theorem. 6

Paper : DSE-2.3

## ( NUMBER THEORY )

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. (a) State Goldbach conjecture. 1
- (b) Prove that if  $a \equiv b \pmod{n}$  and  $m|n$ , then  $a \equiv b \pmod{m}$ . 2
- (c) Using Fermat's theorem, find the remainder when  $2 \times 26!$  is divided by 29. 3
- (d) Solve the following simultaneous congruences : 4
- $$\begin{aligned} x &\equiv 5 \pmod{6} \\ x &\equiv 4 \pmod{11} \\ x &\equiv 3 \pmod{17} \end{aligned}$$
2. Answer any two of the following :  $5 \times 2 = 10$
- (a) Determine all solutions in the positive integers of the Diophantine equation  $18x + 5y = 48$
- (b) Prove that  $53^{103} + 103^{53}$  is divisible by 39.
- (c) State and prove Wilson's theorem.

3. (a) Write the value of  $\sigma(p)$ , where  $p$  is a prime. 1
- (b) Write a reduced set of residues modulo 12. 1
- (c) For any positive integer  $n$ , show that  $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$  2
- (d) Verify that 1000! terminates in 249 zeros. 3
- (e) Prove that if  $n$  is an odd integer, then  $\phi(2n) = \phi(n)$ . 2
- (f) What is a multiplicative function? Prove that if  $f$  is a multiplicative function and  $f(x) \neq 0$  for some  $n \in \mathbb{N}$ , then  $f(1) = 1$ .  $1+2=3$
- (g) Define Dirichlet product of two functions and prove that it is commutative.  $1+2=3$
- (h) For any integer  $n > 1$ , prove that  $n^{\frac{\tau(n)}{2}} = \prod_{d|n} d$  3
4. Answer any three of the following :  $4 \times 3 = 12$
- (a) If  $n \geq 1$  and  $\gcd(a, n) = 1$ , then prove that  $a^{\phi(n)} \equiv 1 \pmod{n}$ .
- (b) For each positive integer  $n \geq 1$ , prove that  $n = \sum_{d|n} \phi(d)$ .

- (c) If  $f$  is a multiplicative function and

$$f(n) = \sum_{d|n} F(d)$$

then by using Möbius inversion formula prove that  $F$  is also multiplicative.

- (d) Use Euler's theorem to establish that 51 divides  $10^{32n+9} - 7$ .

5. (a) Find the order of 2 modulo 17. 1  
 (b) Define primitive Pythagorean triple. Find three different Pythagorean triples, not necessarily primitive, of the form  $16, y, z$ . 1+2=3

Or

Prove that if  $p$  is an odd prime, then  $p^2$  has a primitive root.

- (c) Prove that if  $F_n = 2^{2^n} + 1$ ,  $n > 1$ , is a prime, then 2 is not a primitive root of  $F_n$ . 4

Or

If  $\gcd(m, n) = 1$ , where  $m > 2$  and  $n > 2$ , then show that the integer  $mn$  has no primitive roots.

- (d) Solve the following quadratic congruence : 4  
 $3x^2 + 9x + 7 \equiv 0 \pmod{13}$

- (e) Show that 7 and 18 are the only incongruent solutions of  $x^2 \equiv -1 \pmod{5^2}$  4

- (f) Define the Legendre symbol  $(a/p)$ , where  $p$  is a prime and  $(a, p) = 1$ . Prove that if  $a \equiv b \pmod{p}$ , then  $(a/p) = (b/p)$ . 4

6. Answer any two of the following : 5×2=10

- (a) Let the integer  $a$  has order  $k$  modulo  $n$ . Then show that  $a^h \equiv 1 \pmod{n}$  if and only if  $k|h$ .

- (b) Prove that if  $n$  has a primitive root, then it has exactly  $\phi(\phi(n))$  of them.

- (c) Prove that if  $p$  is a prime number and  $d|p-1$ , then the congruence  $x^d - 1 \equiv 0 \pmod{p}$  has exactly  $d$  solutions.

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Paper : DSE-2.4

( BIOMATHEMATICS )

Full Marks : 80

Pass Marks : 32

Time : 3 hours

UNIT—I

1. Answer any two of the following :  $7\frac{1}{2} \times 2 = 15$

(a) A population is originally 100 individuals, but because of the combined effects of births and deaths, it triples each hour.

(i) Make a table of population size for  $t=0$  to 5, where  $t$  is measured in hours.

(ii) Give two equations modelling the population growth by first expressing  $P_{t+1}$  in terms of  $P_t$  and then expressing  $\Delta P$  in terms of  $P_t$ .

(iii) What can you say about the birth-rate and death rate for this population?

(b) In the early stages of the development of a frog embryo, cell division occurs at a fairly regular rate. Suppose you observe that all cells divide, and hence the

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number of cells double, roughly every half-hour.

(i) Write down an equation modelling this situation. You should specify how much real-world time is required by an increment of 1 in  $t$  and what the initial number of cells is.

(ii) Produce a table and graph of the number of cells as a function of  $t$ .

(c) Obtain a simple prey-predator model explaining in detail the assumptions taken. Also find the equilibrium positions.

UNIT—II

2. Answer any two of the following :  $7\frac{1}{2} \times 2 = 15$

(a) Consider the SI epidemic model. If the contact rate is 0.001 and the number of susceptible is 2000 initially, determine—

(i) the number of susceptible left after 3 weeks;

(ii) the density of susceptible when the rate of appearance of new cases is a maximum;

- (iii) the time (in weeks) at which the rate of appearance of new cases is a maximum;
- (iv) the maximum rate of appearance of new cases.
- (b) In an SIS model, if the infection is spread only by a constant number of carriers, then show that

$$I(t) = \left( I_0 - \frac{\alpha CN}{\alpha C + \beta} \right) e^{-(\alpha C + \beta)t} + \frac{\alpha CN}{\alpha C + \beta}$$

where  $I$  and  $C$  are the numbers of infectives and carriers;  $N$  is total population;  $\alpha$  and  $\beta$  are contact rate and susceptible rate respectively;  $I_0$  is the infectives at  $t = 0$ .

- (c) Let  $x$  and  $y$  respectively denote the proportion of susceptibles and carriers in a population. Suppose the carriers are identified and removed from the population at a rate  $\beta$ , so that  $\frac{dy}{dt} = \beta y$ . Suppose also that the disease spreads at a rate proportional to the product of  $x$  and  $y$ , thus

$$\frac{dx}{dt} = -\alpha xy$$

- (i) Determine the proportions of carriers at any time  $t$ , where  $y(0) = y_0$ .

- (ii) Use (i) to find the susceptibles at time  $t$ , where  $x(0) = x_0$ .
- (iii) Find the proportion of population that escapes the epidemic.

### UNIT—III

3. Answer any *two* of the following :  $7\frac{1}{2} \times 2 = 15$

- (a) Consider the competition models for two species with populations  $N_1$  and  $N_2$

$$\frac{dN_1}{dt} = r_1 N_1 \left( 1 - \frac{N_1}{K_1} - b_{12} \frac{N_2}{K_1} \right)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left( 1 - b_{21} \frac{N_1}{K_2} \right)$$

where only one species  $N_1$  has limited carrying capacity. Investigate their stability and sketch the phase plane trajectories.

[Here,  $K_1, K_2$  are carrying capacities;  $r_1, r_2$  are linear birthrates of the populations  $N_1$  and  $N_2$  respectively;  $b_{12}, b_{21}$  measure the competitive effect of  $N_2$  on  $N_1$  and  $N_1$  on  $N_2$  respectively.]

$$4 + 3\frac{1}{2} = 7\frac{1}{2}$$

- (b) What is Routh-Hurwitz criteria? Explain with reference to multiple species communities.

$$2 + 5\frac{1}{2} = 7\frac{1}{2}$$

- (c) Discuss bifurcation and limit cycle with respect to any biological model.

## UNIT—IV

4. Answer any two of the following :  $7\frac{1}{2} \times 2 = 15$

- (a) Write a short note on any one of the following :

- (i) One-species model with diffusion  
(ii) Two-species model with diffusion

- (b) For a blood vessel of constant radius  $R$ , length  $L$  and driving force  $P = p_1 - p_2$ , show that the average velocity of the flow is equal to half of the maximum velocity and the resistance is proportional to  $\frac{L}{R^4}$ .

- (c) Consider arterial blood viscosity  $\mu = 0.027$  poise.

If the length of the artery is 2 cm, and radius  $8 \times 10^{-3}$  cm and  $P = p_1 - p_2 = 4 \times 10^3$  dynes/cm<sup>2</sup>, then find—

- (i)  $q_z(r)$  and the maximum peak velocity of blood;

- (ii) the shear stress at the wall.  
(Here,  $q_z$  denotes velocity along  $z$ -axis,  $p_1$  and  $p_2$  denote pressure at two ends of the artery.)

## UNIT—V

5. Answer any two of the following :  $10 \times 2 = 20$

- (a) Let  $D$  &  $d$  and  $W$  &  $w$  respectively denote allele for tall & dwarf and round & wrinkled seeds of peas. Find the outcome of the product  $DdWw \times ddWw$  using Punnett square or using probability. Also find the probability that the progeny of  $DdWw \times ddWw$  is dwarf with round seeds.  $6+4=10$

- (b) Explain in detail the Hardy-Weinberg equilibrium, mentioning the assumptions considered for the equilibrium.

- (c) Compare and contrast the stage structure model with age structure model.

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5 SEM TDC DSE MTH (CBCS)  
2.1/2.2/2.3/2.4 (H)

26P—2500/607