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5 SEM TDC MTMH (CBCS) C 11

2025

(Nov/Dec)

MATHEMATICS

(Core)

Paper : C–11

(Multivariate Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* State the domain of the function

$$f(x, y) = \sqrt{4 - x^2 - y^2}$$ 1

(b) Find $\left(\dfrac{\partial f}{\partial y}\right)_{(1,\, 0)}$ where

$$f(x, y) = x^3 y^2 + e^{xy^2}$$ 2

(c) Show that

$$\underset{(x,\,y)\to(0,\,0)}{\text{Lt}} \frac{2xy}{x^2+y^2}$$

does not exist. 3

(d) Using definition, show that the function

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}; & (x, y) \neq (0, 0) \\ 0 \quad ; & (x, y) = (0, 0) \end{cases}$$

is continuous at the origin. 3

Or

For the function

$$f(x, y) = \begin{cases} \dfrac{x^3+y^3}{x-y}; & x \neq y \\ 0 \quad ; & x = y \end{cases}$$

show that $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist at (0, 0) and are equal.

(e) Show that $f_{xy} = f_{yx}$ where

$$f(x, y) = x \sin y + y \sin x + xy$$ 3

(f) Find the values of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point (π, π, π) for the function

$$\sin(x+y) + \sin(y+z) + \sin(z+x) = 0$$ 4

Or

Find $\left(\frac{dw}{dt}\right)_{t=1}$ where $w = z - \sin xy$ and $x = t,\ y = \log t,\ z = e^{t-1}$.

(g) Find an equation for tangent plane to the surface $z = \tan^{-1}\frac{y}{x}$ at the point $(1, \sqrt{3}, \frac{\pi}{3})$. 4

Or

Find the directional derivative of $f(x, y) = e^{x^2y^2}$ at $P_0(1, -1)$ in the direction towards $Q(2, 3)$.

(h) Find a vector that is normal to the level surface $x^2 + 2xy - yz + 3z^2 = 7$ at the point (1, 1, −1). 2

(i) Determine the stationary points of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ and classify the points. 4

Or

Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

(j) Find div $\vec{F}$ and curl $\vec{F}$ where

$$\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$$ 4

Or

If $\vec{F} = x^2 y\hat{i} + xz\hat{j} + 2yz\hat{k}$, find $\nabla \cdot (\nabla \times \vec{F})$.

2. (a) Evaluate

$$\iint_R x^2 y^5 \, dA$$

where $R : [1, 2; 0, 1]$. 2

(b) Show that

$$\int_0^1 dx \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dy = \int_0^1 dy \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dx$$ 4

3. (a) Define Jacobian of a function of two variables. 1

(b) Evaluate

$$\iint_E \sin\left(\frac{x-y}{x+y}\right) dx\, dy$$

where E is the region bounded by the coordinate axes and $x + y = 1$ in the first quadrant. 4

Or

Use transformations $u = x - y$ and $v = 2x + y$ to evaluate the integral

$$\iint_R (2x^2 - xy - y^2)\, dx\, dy$$

where R is the region bounded by the lines $y = -2x + 4$, $y = -2x + 7$, $y = x - 2$ and $y = x + 1$.

(c) Define flux across a plane curve. 1

(d) Find the value of

$$\int_C \{(x+y)^2 dx + (x^2 - y)\, dy\}$$

taken in the clockwise sense along the closed curve C formed by $y^3 = x^2$ and the chord joining (0, 0) and (1, 1). 4

Or

Evaluate $\int_C f(x, y)\, dS$, where $f(x, y) = x + y$ and C is the circle $x^2 + y^2 = 4$ in the first quadrant from (2, 0) to (0, 2).

(e) State and prove the fundamental theorem of line integrals. 5

Or

Find the value of $\int_C (x^2 y dx + xy^2 dy)$ taken in the clockwise sense along the hexagon whose vertices are $(\pm 3a, 0)$ $(\pm 2a, \pm\sqrt{3}a)$.

4. (a) State and prove Green's theorem. 5

Or

Prove that the line integral

$$\int_C \frac{x\,dy - y\,dx}{x^2 + y^2}$$

taken in the positive direction over any closed contour with the origin inside it, is equal to 2π.

(b) Write one property of surface integral. 1

(c) Evaluate

$$\iint_S x\,dy\,dz + dz\,dx + xz^2\,dx\,dy$$

where S is the outer side of the part of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. 4

Or

Use Stokes' theorem to find the line integral $\int_C x^2 y^3\,dx + dy + z\,dz$.

(d) Prove that the flux of a vector field $\vec{F} = M\hat{i} + N\hat{j} + P\hat{k}$ where M, N, P are functions of x, y and z across a closed piecewise smooth oriented surface S, in the direction of its outward unit normal field $\hat{n}$ is equal to

$$\iiint_D \nabla \cdot \vec{F}\,dV$$

where D is the convex region without holes or bubbles. 5

Or

Use divergence theorem to find the outward flux of $\vec{F}$ across the boundary of the region D, where

$$\vec{F} = (y - x)\hat{i} + (z - y)\hat{j} + (y - x)\hat{k}$$

and D is the cube bounded by the planes $x = \pm 1$, $y = \pm 1$ and $z = \pm 1$.

Or

Evaluate $\iint_D \frac{dA}{y^2 + 1}$; D is the triangle bounded by $x = 2y$, $y = -x$ and $y = 2$.

(e) Reverse the order of integration in the iterated integral

$$\int_0^2 \int_1^{e^x} f(x, y)\,dy\,dx$$ 2

(f) Using double integral, find the area of the region between $y = \cos x$ and $y = \sin x$ over the interval $0 \le x \le \frac{\pi}{4}$. 2

(g) Evaluate $\iiint_B z^2 y e^x \, dV$ where B is the box given by $0 \le x \le 1$, $1 \le y \le 2$, $-1 \le z \le 1$. 2

(h) Compute the volume of the solid bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the surface of the paraboloid $x^2 + y^2 = 3z$. 4

Or

Find the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane $x + \frac{y}{2} + \frac{z}{3} = 1$.

(i) Evaluate the following integral by changing the order of the integration in an appropriate way :

$$\int_0^4 \int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}} \, dx \, dy \, dz$$

4

Or

Evaluate :

$$\int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_{\sec\phi}^2 3\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

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