Total No. of Printed Pages—4

3 SEM TDC PHYH (CBCS) C 5

2025

(Nov/Dec)

PHYSICS

(Core)

Paper : C–5

(Mathematical Physics—II)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×5=5

(a) To satisfy the Dirichlet conditions, a function $f(x)$ should be

(i) single-valued and bounded

(ii) piecewise continuous

(iii) a finite number of extrema

(iv) All of the above

(b) The value of the integral

$$\int_{-\infty}^{+\infty} e^{-\alpha^2} H_m(x) H_n(x)\, dx$$

for $m = n$ is

(i) 0

(ii) 1

(iii) $2^n n! \pi$

(iv) $2^n n! \sqrt{\pi}$

(c) If $L_n(x) = e^x \frac{d^n}{dx^n}(x^n e^{-x})$, then $L_1(x)$ is

(i) 1

(ii) $(1-x)$

(iii) $(2-4x+x^2)$

(iv) x

(d) $erf(\infty)$ is equal to

(i) 0

(ii) 1

(iii) -1

(iv) ∞

(e) If $J = \frac{\partial(u, v)}{\partial(x, y)}$ and $J' = \frac{\partial(x, y)}{\partial(u, v)}$, then JJ' is equal to

(i) 0

(ii) -1

(iii) ∞

(iv) 1

2\. (a) Find the Fourier series for the following functions : 6

$$f(x) = \begin{cases} -k, & \text{when } -\pi < x < 0 \\ k, & \text{when } 0 < x < \pi \end{cases}$$

(b) Find the Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. 5

Or

Expand in Fourier series the function $f(x) = x$ in the interval $-1 < x < 1$.

3\. (a) Prove that

$$\int_{-1}^{+1} [P_n(x)]^2\, dx = \frac{2}{2n+1}$$ 5

(b) Solve any *one* of the following differential equations : 6

(i) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$

(ii) $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2ny = 0$

(c) Prove the following recurrence formulae for $P_n(x)$: 2×2=4

(i) $(n+1)P_n(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$

(ii) $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$

4\. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 3

Or

Express $\int_0^{\frac{\pi}{2}} \sin^p \theta \cos^q \theta \, d\theta$ in the form of beta and gamma functions.

5. (a) If $\Delta x = 0 \cdot 02$, find Δy, where $y = 2x^5 + 3x^2 - 2$ and hence find the relative error of y at $x = 2$. 3

(b) Explain the method of least square fitting. 3

6. (a) Find the solution of Laplace equation either in 3D Cartesian or cylindrical coordinates. 5

(b) The partial differential equation

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{k} \frac{\partial z}{\partial t}$$

where $z = z(x, t)$. What will be the solutions of the above partial differential equation if the constant of separation is (i) positive and (ii) negative? 3

(c) Solve $\frac{\partial^2 z(x, y)}{\partial x \partial y} = x^2 y$ subject to condition $z(x, 0) = x^2$, $z(1, y) = \cos y$. 5

★ ★ ★