Total No. of Printed Pages—8

1 SEM TDC STSH (CBCS) C 2

2021

(Held in January/February, 2022)

STATISTICS

(Core)

Paper : C–2

(Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following alternatives in each question : 1×8=8

(a) If f and g be two functions which are continuous at $x = a$, then the function, which will be continuous at $x = a$, is

(i) $f \pm g$

(ii) $f \cdot g$

(iii) $k \cdot f$ (k is a constant)

(iv) All of the above

(b) If $\lim_{x\to a} \phi(x) = 0$ and $\lim_{x\to a} \psi(x) = \infty$, then $\lim_{x\to a} \phi(x) \cdot \psi(x)$ is said to assume indeterminate form

(i) $\frac{0}{0}$

(ii) $\frac{\infty}{\infty}$

(iii) $0 \times \infty$

(iv) None of the above

(c) If $y = e^{-2x}$, then y_3 is

(i) $8e^{2x}$

(ii) $-8e^{-2x}$

(iii) $4e^{2x}$

(iv) None of the above

(d) The value of $\Gamma(n+1)$ is

(i) $(n+1)!$

(ii) $(n-1)\,\Gamma(n-1)$

(iii) $n\,\Gamma n$

(iv) None of the above

(e) The value of $\int_1^2 \int_0^{3y} y\,dy\,dx$ is

(i) 3

(ii) 5

(iii) 7

(iv) 9

(f) The integrating factor of the equation

$$\frac{dy}{dx} + \frac{y}{x} = y^2$$

is

(i) $\log(1/x)$

(ii) $1/x$

(iii) $\log x$

(iv) None of the above

(g) If $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$, then its particular integral is

(i) e^{5x}

(ii) $\frac{e^{5x}}{12}$

(iii) $\frac{e^{2x}}{10}$

(iv) None of the above

(h) If $z = ax + by + ab$, then the partial differential equation is

(i) $z = x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x}$

(ii) $z = x\frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right) + y\frac{\partial z}{\partial x}$

(iii) $z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)$

(iv) None of the above

2. Answer the following questions :

(a) Examine the continuity of the following function at $x = 2$: 3

$$f(x) = \begin{cases} (x^2/a) - a & , \text{ for } 0 < x < a \\ 0 & , \text{ for } x = 0 \\ a - (a^3/x^3) & , \text{ for } x > 0 \end{cases}$$

(b) If

$$u_1 = \frac{x_2 x_3}{x_1},\ u_2 = \frac{x_3 x_1}{x_2},\ u_3 = \frac{x_1 x_2}{x_3}$$

then show that, $J(u_1, u_2, u_3) = 4$. 3

(c) Evaluate : 4

$$\int_0^{\pi/2} \log \sin x \, dx$$

(d) Solve : 4

$$(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$$

(e) Find the partial differential equation by eliminating a and b from

$$z = ax + by + a^2 + b^2$$ 2

3. Answer any *three* questions : 7×3=21

(a) Define right-hand limit and left-hand limit of a function. Find

$$\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$$

(b) What do you mean by partial differentiation? If

$$u = \frac{1}{\sqrt{x^2 + y^2 + z^2}},\ x^2 + y^2 + z^2 \neq 0$$

then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

(c) Give the statement of L-Hospital's rule. Evaluate

$$\lim_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$$

(d) Find the nth derivative of $y = e^{2x}x^3$.

(e) What is a homogeneious function? State and prove Euler's theorem on homogeneous function.

(f) If $u = \dfrac{x^3 - y^3}{x^2 + y^2}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$$

(g) Find for what value(s) of x the function

$$f(x) = 41 - 72x - 18x^2$$

attains its maximum.

4. Answer any *two* questions : 7×2=14

(a) Evaluate :

$$\int_0^2 \int_0^{\sqrt{4+x^2}} \frac{dx\,dy}{4+x^2+y^2}$$

(b) Using the properties of definite integrals, show that

$$\int_0^{\pi/2} \frac{\sin x - \cos x}{\sin x + \cos x} dx = 0$$

(c) Write different forms of beta function. Show that beta function is symmetric.

(d) Show that

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta \, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$$

where, $p > -1$, $q > -1$ and hence deduce that

$$\int_0^2 x^4 (8 - x^3)^{-1/3} dx = \frac{16}{3}\beta\left(\frac{5}{3}, \frac{2}{3}\right)$$

(e) Prove that

$$\Gamma(1+n)\Gamma(1-n) = \frac{n\pi}{\sin n\pi}$$

and hence find the value of $\Gamma\frac{1}{2}$.

5. Answer any *two* questions : 7×2=14

(a) Write the general form of a first-order and first-degree differential equation. Discuss the working rule for solving a differential equation of the form

$$f_1(x)\,dx = f_2(y)\,dy$$

(b) What is the working rule for solving an exact diferential equation? Solve

$$(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$$

(c) What is an integrating factor? Solve the following differential equation :

$$(xy^2 + 2x^2y^3)dx + (x^2y - x^3y^2)dy = 0$$

(d) What is a linear differential equation? Solve

$$\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 0$$

(e) Write the integrability condition of total differential equation. Solve

$$(y + z)dx + dy + dz = 0$$

6. Answer any *one* question : 7

(a) Distinguish between partial and ordinary differential equations. Solve the following differential equation by direct integration method :

$$X\frac{\partial^2 Z}{\partial X^2} = \frac{\partial Z}{\partial X}$$

(b) Discuss the Lagrange's method to solve partial differential equation of the form

$$Pp + Qq = R$$

Solve the following differential equation by Lagrange's method :

$$(Y+Z)p - (X+Z)q = X - Y$$

(c) What do you mean by linear homogeneous partial differential equation with constant coefficient? Solve the following :

$$\frac{\partial^2 z}{\partial x^2} = a^2\left(\frac{\partial^2 z}{\partial y^2}\right)$$

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1 SEM TDC STSH (CBCS) C 2

2021

(Held in January/February, 2022)

STATISTICS

(Core)

Paper : C–2

(Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following alternatives in each question : 1×8=8

(a) If f and g be two functions which are continuous at $x = a$, then the function, which will be continuous at $x = a$, is

(i) $f \pm g$

(ii) $f \cdot g$

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(Continued)

(e) The value of $\int_1^2 \int_0^{3y} y\,dy\,dx$ is

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(Turn Over)

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$$\frac{\partial^2 z}{\partial x^2}=a^2\left(\frac{\partial^2 z}{\partial y^2}\right)$$

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Total No. of Printed Pages—4

1 SEM TDC STSH (CBCS) C 1

2021

(Held in January/February, 2022)

STATISTICS

(Core)

Paper : C–1

(Descriptive Statistics)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer of the following : $1 \times 5 = 5$

(a) With the help of ogive, one can determine

(i) median

(ii) deciles

(iii) quartiles

(iv) All of the above

(b) Arithmetic mean, geometric mean and harmonic mean in any series are equal when

   (i) the distribution is symmetric

   (ii) all the values are same

   (iii) the distribution is positively skewed

   (iv) the distribution is unimodal

(c) The sum of squares of deviation of observations is least when measured from

   (i) median

   (ii) origin

   (iii) mode

   (iv) None of the above

(d) The coefficient of correlation

   (i) cannot be positive

   (ii) cannot be negative

   (iii) is always positive

   (iv) can be both positive as well as negative

(e) Index for base period is always taken as

   (i) 1

   (ii) 100

   (iii) 1000

   (iv) 0

2. Answer the following questions in brief : 2×5=10

(a) Differentiate between primary data and secondary data.

(b) For two values x_1 and x_2, prove that $AM \times HM \geq (GM)^2$.

(c) Write down the Bowley's formula for measuring skewness.

(d) Explain why there are two lines of regression.

(e) What do you mean by chain-based index number?

3. (a) What are the different measures of scales used in statistics? Explain with suitable example. 7

Or

(b) What do you mean by cumulative frequency curve or ogive? What are the different types of ogive? Explain their uses. 2+2+3=7

4. Anwer any *two* questions of the following : 7×2=14

(a) Compare mean, median and mode as measures of location of a distribution. Prove that the sum of the squares of the deviations of observations about mean is the least. 4+3=7

(b) What is standard deviation? Explain its superiority over other measures of dispersion. Show that for any distribution, the standard deviation is not less than the mean deviation from the mean. 2+2+3=7

(c) Define raw moments and central moments. Obtain the relation between the central moments of order r in terms of the raw moments. 2+5=7

5. (a) Define rank correlation. Deduce Spearman's formula for rank correlation coefficient. 2+5=7

Or

(b) Define multiple correlation. Show that the multiple correlation coefficient $R_{1\cdot 23}$ in usual notation is given by

$$R^2_{1\cdot 23} = 1 - \frac{\omega}{\omega_{11}}$$ 2+5=7

6. (a) Explain briefly time-reversal test and factor-reversal test of index number. Show that Fisher's index number formula satisfies both the time-reversal test and factor-reversal test. 2+5=7

Or

(b) What do you mean by consumer price index? Discuss the various steps in the construction of consumer price index. 2+5=7

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1 SEM TDC STSH (CBCS) C 1

2021

(Held in January/February, 2022)

STATISTICS

(Core)

Paper : C–1

(Descriptive Statistics)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer of the following : 1×5=5

(a) With the help of ogive, one can determine

(i) median

(ii) deciles

(iii) quartiles

(iv) All of the above

(b) Arithmetic mean, geometric mean and harmonic mean in any series are equal when

(i) the distribution is symmetric
(ii) all the values are same
(iii) the distribution is positively skewed
(iv) the distribution is unimodal

(c) The sum of squares of deviation of observations is least when measured from

(i) median
(ii) origin
(iii) mode
(iv) None of the above

(d) The coefficient of correlation

(i) cannot be positive
(ii) cannot be negative
(iii) is always positive
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(iii) 1000
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