2 SEM TDC STSH (CBCS) C 3 (N/O)

2025

(May)

STATISTICS

(Core)

Paper : C–3

(Probability and Probability Distribution)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following alternatives : 1×5=5

(a) The probability of the intersection of two mutually exclusive events is always

(i) infinity

(ii) zero

(iii) one

(iv) None of the above

(b) If X and Y are two random variables, then the covariance between them is defined as

(i) $\text{cov}(X, Y) = 0$

(ii) $\text{cov}(X, Y) = E(XY) - E(X)E(Y)$

(iii) $\text{cov}(X, Y) = E(XY) + E(X)E(Y)$

(iv) $\text{cov}(X, Y) = E(XY)$

(c) If X is a random variable having its probability function $f(x)$, then $E(X)$ is called

(i) arithmetic mean

(ii) geometric mean

(iii) harmonic mean

(iv) first quartile

(d) For binomial probability distribution

(i) mean = variance

(ii) mean > variance

(iii) mean < variance

(iv) None of the above

(e) The ratio of two independent normal variates is a/an

(i) normal variate

(ii) Cauchy variate

(iii) exponential variate

(iv) beta variate

2. Answer the following questions : 2×5=10

(a) Explain briefly the axiomatic approach to probability.

(b) Two fair dice are thrown simultaneously. What is the probability that the sum of numbers on the dice exceeds 8?

(c) Define discrete and continuous random variables.

(d) Prove that if X, Y are independent random variables, then

$$E(XY) = E(X)\,E(Y)$$

(e) Define characteristic function and give its importance.

3. (a) State and prove the addition theorem of probability. 2+4=6

Or

(b) In 2026, there will be three candidates for the position of principal of a college— Mr. X, Mr. Y and Mr. Z, whose chances of getting the appointment are in the proportion 4 : 2 : 3 respectively. The chance that Mr. X if selected will introduce coeducation in the college is 0·3. The chances of Mr. Y and Mr. Z doing the same are 0·5 and 0·8. What is the chance that there will be coeducation in the college in 2026? 6

4. (a) Define p.m.f. of a random variable. A random variable X takes the following values with probabilities :

X	0	1	2	3	4	5	6	7
$P(X)$	0	$2k$	$3k$	k	$2k$	k^2	$7k^2$	$2k^2+k$

(i) Find k.

(ii) Find $P(X<5)$.

(iii) Find $P(X \geq 5)$. 2+6=8

Or

(b) Define p.d.f. of a random variable. The p.d.f. of a random variable is

$$f(x)=\begin{cases} 0 & , \ x \leq 2 \\ \frac{1}{18}(3+2x) & , \ 2<x<4 \\ 0 & , \ x \geq 4 \end{cases}$$

Determine (i) $\int_{-\infty}^{\infty} f(x)\,dx$ and (ii) $E(X)$. 2+3+3=8

5. (a) Define mathematical expectation of a discrete and continuous random variable. A continuous random variable has a p.d.f.

$$f(x)=kx^2e^{-x} \ ; \ x>0$$

Find k, $E(X)$ and $V(X)$. 2+2+2+2=8

Or

(b) (i) Prove that the m.g.f. of the sum of independent variables is equal to the product of their respective m.g.f.s. 4

(ii) Explain the properties of distribution function with examples. 4

6. *(a)* Define binomial distribution. Obtain the m.g.f. of binomial distribution. Hence or otherwise find its mean and variance. 2+3+4=9

Or

(b) Define Poisson distribution. Give some examples of Poisson distribution. Derive Poisson distribution as a limiting case of binomial distribution. 2+2+5=9

7. *(a)* Define exponential distribution. Obtain its m.g.f. and hence find its mean and variance. 2+3+4=9

Or

(b) Define gamma distribution with two parameters. State and prove the additive property of gamma distribution. 2+2+5=9

(Continued)

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

1. Choose the correct answer from the following alternatives : 1×5=5

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(iv) None of the above

(b) If X and Y are two random variables, then the covariance between them is defined as

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(Turn Over)

(c) If X is a random variable having its probability function $f(x)$, then $E(X)$ is called

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(i) Find k.

(ii) Find $P(X<5)$. 2+2+2=6

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(b) Define p.d.f. of a random variable. The p.d.f. of a random variable is

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2+2+2=6

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$$f(x)=kx^2e^{-x}\ ; \ x>0$$

Find k and $E(X)$. 2+2+2=6

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(ii) Explain the properties of distribution function with examples. 3

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