Total No. of Printed Pages—9

3 SEM TDC STSH (CBCS) C 5 (N/O)

2023

(Nov/Dec)

STATISTICS

(Core)

Paper : C–5

(Sampling Distribution)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following : 1×5=5

   (a) If X is a continuous random variable with mean μ and variance σ^2, then for any positive number K

   $$P\{|X-\mu| \geq K\sigma\} \leq \frac{1}{K^2}$$

   is known as

   (i) Chebychev's inequality

   (ii) weak law of large number

   (iii) strong law of large number

   (iv) None of the above

(b) The SE of mean of a random sample of size n from a population with variance 4 is

(i) 2

(ii) $\frac{2}{n}$

(iii) $\frac{2}{\sqrt{n}}$

(iv) $\frac{n}{2}$

(c) The range of χ^2-variate is

(i) $-\infty$ to ∞

(ii) 0 to ∞

(iii) 0 to 1

(iv) $-\infty$ to 0

(d) The area of critical region depends on

(i) the size of type–I error

(ii) the size of type–II error

(iii) the value of the statistic

(iv) the number of observations

(e) The variance of Student's t-distribution is

(i) n

(ii) $2n$

(iii) $\frac{\nu}{\nu-2}$, where ν = degrees of freedom

(iv) None of the above

2. Answer the following questions : 2×5=10

(a) Define convergence in probability.

(b) What are type–I error and type–II error?

(c) What are the uses of order statistics?

(d) Define F-statistic. Write down its probability density function.

(e) State additive property of χ^2-variates.

3. (a) Obtain the sampling distribution of mean of a random sample drawn from a normal population with mean μ and variance σ^2. 6

Or

(b) Define parameter and statistic. Explain the importance of sampling distribution of a statistic in Statistics. 2+4=6

4. Describe, how you would test the difference of two means in large samples. 4

5. (a) Define rth order statistic $X_{(r)}$. Obtain the joint probability density function of $X_{(r)}$ and $X_{(s)}$, $r < s$ in a random sample of size n from a population with continuous distribution function $F(x)$. Hence deduce the probability density function of sample range $W = X_{(n)} - X_{(1)}$. 7

Or

(b) State Chebychev's inequality. If X is the number scored in a throw of a fair die, show that the Chebychev's inequality gives $P\{|X - \mu| > 2 \cdot 5\} < 0 \cdot 47$, where μ is the mean of X, while the actual probability is zero. 2+5=7

6. (a) Define χ^2-statistic. Derive the probability density function of χ^2 with n degrees of freedom using moment-generating function. 6

Or

(b) Show that χ^2-distribution approaches to normality when $n \to \infty$.

7. (a) Describe the χ^2-test of goodness of fit. 5

Or

(b) Find the mode of χ^2-distribution with n degrees of freedom.

8. Answer any *two* of the following questions : 6×2=12

(a) Write down the density function of t-distribution. Describe the chief features of t-distribution curve.

(b) Derive the distribution of F.

(c) Find the variance of the t-distribution with n degrees of freedom $(n > 2)$.

(d) Obtain the mode of F-distribution with (n_1, n_2) degrees of freedom and show that it lies between 0 and 1.

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

1. Choose the correct answer from the following : 1×5=5

(a) If X is a continuous random variable with mean μ and variance σ^2, then for any positive number K

$$P\{|X-\mu| \geq K\sigma\} \leq \frac{1}{K^2}$$

is known as

(i) Chebychev's inequality

(ii) weak law of large number

(iii) strong law of large number

(iv) None of the above

(b) The SE of mean of a random sample of size n from a population with variance 4 is

(i) 2

(ii) $\frac{2}{n}$

(iii) $\frac{2}{\sqrt{n}}$

(iv) $\frac{n}{2}$

(c) The range of χ^2-variate is

(i) $-\infty$ to ∞

(ii) 0 to ∞

(iii) 0 to 1

(iv) $-\infty$ to 0

(d) The area of critical region depends on

(i) the size of type–I error

(ii) the size of type–II error

(iii) the value of the statistic

(iv) the number of observations

(e) The variance of Student's t-distribution is

(i) n

(ii) $2n$

(iii) $\frac{\nu}{\nu-2}$, where ν = degrees of freedom

(iv) None of the above

2. Answer the following questions : 2×5=10

(a) Define convergence in probability.

(b) What are type–I error and type–II error?

(c) What are the uses of order statistics?

(d) Define F-statistic. Write down its probability density function.

(e) State additive property of χ^2-variates.

3. (a) Obtain the sampling distribution of mean of a random sample drawn from a normal population with mean μ and variance σ^2. 5

Or

(b) Define parameter and statistic. Explain the importance of sampling distribution of a statistic in Statistics. 2+3=5

4. Describe, how you would test the difference of two means in large samples. 3

5. (a) Define rth order statistic $X_{(r)}$. Obtain the joint probability density function of $X_{(r)}$ and $X_{(s)}$, $r < s$ in a random sample of size n from a population with continuous distribution function $F(x)$. Hence deduce the probability density function of sample range $W = X_{(n)} - X_{(1)}$. 6

Or

(b) State Chebychev's inequality. If X is the number scored in a throw of a fair die, show that the Chebychev's inequality gives $P\{|X-\mu| \geq 2 \cdot 5\} < 0 \cdot 47$, where μ is the mean of X, while the actual probability is zero. 2+4=6

6. (a) Define χ^2-statistic. Derive the probability density function of χ^2 with n degrees of freedom using moment-generating function. 5

Or

(b) Show that χ^2-distribution approaches to normality when $n \to \infty$.

7. (a) Describe the χ^2-test of goodness of fit. 4

Or

(b) Find the mode of χ^2-distribution with n degrees of freedom.

8. Answer any *two* of the following questions : 6×2=12

(a) Write down the density function of t-distribution. Describe the chief features of t-distribution curve.

(b) Derive the distribution of F.

(c) Find the variance of the t-distribution with n degrees of freedom $(n > 2)$.

(d) Obtain the mode of F-distribution with (n_1, n_2) degrees of freedom and show that it lies between 0 and 1.

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Total No. of Printed Pages—11

3 SEM TDC STSH (CBCS) C 6 (N/O)

2023

(Nov/Dec)

STATISTICS

(Core)

Paper : C–6

(Survey Sampling and Indian Official Statistics)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer of each question from given alternatives : 1×5=5

(a) The difference between sample estimate and population parameter is known as

(i) sampling error

(ii) non-sampling error

(iii) formula error

(iv) human error

(b) The standard error of the mean of the sample of size n, which is drawn from the population with variance σ^2 is

(i) σ^2 / n

(ii) σ / n

(iii) σ^2 / n^2

(iv) $\sigma / \sqrt{n}$

(c) In stratified random sampling, optimum allocation of sample size in each stratum is done to

(i) minimize the variance for fixed sample size

(ii) maximize the precision for fixed cost

(iii) minimize the total cost for fixed desired precision

(iv) All of the above

(d) Linear regression estimate is used when the lines of regression pass through the

(i) origin

(ii) any arbitrary point

(iii) point on the X-axis

(iv) point on the Y-axis

(e) In SRSWOR for attribute, with usual notation

(i) $\text{var}(p) = \frac{N-n}{N-1} \cdot \frac{PQ}{n}$

(ii) $\text{var}(p) = \frac{N-n}{N} \cdot \frac{pq}{n-1}$

(iii) $\text{var}(p) = \frac{N-n}{n-1} \cdot \frac{PQ}{n}$

(iv) $\text{var}(p) = \frac{N-n}{N-1} \cdot \frac{PQ}{N}$

2. Answer the following : $2 \times 5 = 10$

(a) What do you mean by non-sampling errors?

(b) Define simple random sampling with replacement and without replacement from a finite population.

(c) In what way, linear regression estimate differs from ratio estimate?

(d) How to draw a stratified random sample?

(e) Name two statistical offices of Government of India.

3. Answer any *one* of the following :

(a) (i) What is a sample survey? In what respects, it is superior to a census survey? 6

(ii) Discuss briefly the basic principles of a sample survey. 6

(b) Consider a population of 6 units with values 1, 2, 3, 4, 5, 6. Write down all possible samples of 2 (without replacement) from the population and verify that sample mean is an unbiased estimate of the population mean. Calculate the sampling variances of sample mean by 'with replacement' and 'without replacement' methods and compare these two. 12

(c) In a population of N units, the number of units possessing a certain characteristic is A and in a simple random sample of size n from it, the number of units possessing that characteristic is a. If

$$P = \frac{A}{N},\ p = \frac{a}{n};\ Q = 1 - P \text{ and } q = 1 - p$$

prove that p is an unbiased estimate of P and $\text{var}(p) = \frac{N-n}{N-1} \cdot \frac{PQ}{n}$.

Also prove that an unbiased estimate of var (p) is

$$v(p) = \frac{N-n}{N-1} \cdot \frac{pq}{N}$$ 12

(*Continued*)

4. (a) A population of N units is divided into k strata, there being N_i units in the ith stratum. n_i units are drawn at random without replacement from the ith stratum, the samples from different strata are independent. State the best linear unbiased estimator for the population mean and obtain its variance. Considering a linear cost function $C = a_0 + \sum_{i=1}^{k} C_i n_i$, a_0 being the overhead cost and C_i the cost per unit for the ith stratum. Obtain the optimum values of n_i $(i = 1, 2, \cdots, k)$, such that for the given cost the variance is minimized. 11

Or

(b) What is stratified random sampling? When will you use stratified random sampling? Describe the methods used to fix the number of units to be selected from each stratum. Obtain $V(\overline{y}_{st})$ and obtain the results for the particular cases when—

(i) $\frac{n_h}{N_h}$ is negligible;

(ii) $n_h = \frac{nN_h}{N}$;

(iii) the sampling is proportional and variance in all strata have the same value. 11

(*Turn Over*)

5. *(a)* What is a ratio estimate? Show that the usual ratio estimate is biased and obtain the magnitude of the bias. 4+3+3=10

Or

(b) *(i)* Prove that in systematic sampling positive intraclass correlation coefficient between units of the same systematic sample inflates the variance of the systematic sample mean. 5

(ii) Write a note on cluster sampling. 5

6. *(a)* Explain the role and importance of National Sample Survey Organization in providing data on Indian economy. 7

Or

(b) Write a brief note on present official statistical system in India. What is the role of Ministry of Statistics and Programme Implementation of Government of India?

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

1. Choose the correct answer from the following alternatives in each question : 1×5=5

(a) Sampling is inevitable in the situation

(i) blood test of a person

(ii) when the population is infinite

(iii) testing of life of dry battery cells

(iv) All of the above

(b) The number of all possible samples of size 2 from a population of size 10 without replacement is

(i) 5

(ii) 45

(iii) 100

(iv) None of the above

(c) Stratified sampling comes under the category of

(i) unrestricted sampling

(ii) subjective sampling

(iii) purposive sampling

(iv) restricted sampling

(d) The greatest drawback of systematic sampling is that

(i) one requires a large sample

(ii) data are not easily accessible

(iii) no single reliable formula for standard error of mean is available

(iv) None of the above

(e) In cluster sampling, it is desirable to have

(i) as great a heterogeneity as possible within cluster

(ii) as small a difference as possible between clusters

(iii) Both (i) and (ii)

(iv) None of the above

2. Answer the following in brief : 2×5=10

(a) What is meant by sampling error?

(b) What is lottery method for selecting a simple random sample?

(c) What is proportional allocation in stratified random sampling?

(d) Name two Government of India's principal publications containing statistical data on various factors of population.

(e) What are the points that should be born in mind when we use cluster sampling?

3. (a) Distinguish between sample survey and complete enumeration. Describe briefly the advantages of carrying out a sample survey in preference to a complete enumeration. Under what circumstances can complete enumeration be recommended in preference to a sample survey? 2+5+3=10

Or

(b) What do you mean by simple random sampling with replacement and without replacement from a finite population? Prove that in simple random sampling without replacement the sample mean is an unbiased estimate of the population mean. Under what circumstances can we consider simple random sampling without replacement more efficient than simple random sampling with replacement and how? 4+3+3=10

4. (a) Describe the procedure of stratified random sampling. Mention two commonly used stratifying factors. Describe the methods used to fix the number of units to be selected from each stratum. Prove that for stratified random sampling

$$\bar{y}_{st} = \frac{1}{N}\sum_{i=1}^{k} N_i \bar{y}_{ni}$$

is an unbiased estimator of the population mean $\bar{y}_N$. 3+1+3+3=10

Or

(b) Explain the method of systematic sampling. Discuss its advantage and disadvantage. Obtain the variance of the estimated mean. 3+3+4=10

5. (a) What do you mean by ratio estimate? With usual notation, derive the approximate variance of the ratio estimates $\hat{R}$ and $\hat{Y}_R$. 2+5+2=9

Or

(b) Describe cluster sampling. A simple random sample of n clusters is drawn without replacement from a population of N clusters, each containing as elements. Give an unbiased estimate of the mean square between elements of the population. 3+6=9

6. (a) What is CSO? Write an explanatory note on objective and activities of CSO. 2+4=6

Or

(b) What is meant by NSSO? Mention some functions of NSSO. Under what ministry does NSSO function? 6

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3 SEM TDC STSH (CBCS) C 7 (N/O)

2023

(Nov/Dec)

STATISTICS

(Core)

Paper : C–7

(Mathematical Analysis)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following alternatives : 1×6=6

 (a) The set N of natural numbers is

  (i) bounded above

  (ii) bounded below

  (iii) Both *(i)* and *(ii)*

  (iv) None of the above

(b) The series Σu_n of positive terms is convergent or divergent as

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} > 1 \text{ or } < 1$$

Then the test is known as

(i) comparison test

(ii) Raabe's test

(iii) Cauchy's condensation test

(iv) D'Alembert's test

(c) The series $\Sigma u_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ is

(i) absolute convergent

(ii) conditional convergent

(iii) Both (i) and (ii)

(iv) None of the above

(d) A function f is said to be continuous from the left at c, if

(i) $\lim_{x\to c+0} f(x) = f(c)$

(ii) $\lim_{x\to c+0} f(x) = f(0)$

(iii) $\lim_{x\to c-0} f(x) = f(c)$

(iv) None of the above

(e) The nth divided difference of a polynomial of nth degree is

(i) always zero

(ii) always equal to n

(iii) always constant

(iv) not defined

(f) Lagrange's formula is useful for

(i) interpolation

(ii) extrapolation

(iii) inverse interpolation

(iv) All of the above

2. Answer the following questions in brief : 2×6=12

(a) Define derived set with examples.

(b) State the Cauchy's general principle of convergence.

(c) Define Raabe's test.

(d) State the Taylor's theorem with the remainder in Cauchy's form.

(e) State two properties of divided difference.

(f) Define transcendental equation with examples.

3. Answer any *two* of the following questions : 5×2=10

(a) Show that the set of real numbers forms a complete ordered field.

(b) Prove that a set is closed iff its complement is open.

(c) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}}\right]=1$$

4. (a) (i) Define infinite series. Under what condition a geometric series is convergent? Show that the series

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$$

is not convergent. 1+1+3=5

(ii) Define Cauchy's nth root test. By virtue of D'Alembert's ratio test, test whether the series

$$\frac{1^2\cdot 2^2}{1!}+\frac{2^2\cdot 3^2}{2!}+\frac{3^2\cdot 4^2}{3!}+\cdots$$

is convergent or divergent. 2+4=6

Or

(b) (i) Give the statement of Cauchy's condensation test. Test the convergence of the following series using Raabe's test : 1+4=5

$$\frac{\alpha}{\beta}+\frac{1+\alpha}{1+\beta}+\frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)}+\cdots$$

(ii) Show that every absolutely convergent series is convergent. Show that the series

$$\frac{\log 2}{2}+\frac{\log 3}{3}+\frac{\log 4}{4}+\cdots\infty$$

is divergent. 3+3=6

5. (a) Define uniform continuity of a function. Find the values of a and b so that $f(x)$ may be differentiable at $x=1$, where

$$f(x)=\begin{cases}x^2+3x+a\ ; & \text{if } x\le 1\\ bx+2\ ; & \text{if } x>1\end{cases}$$

2+5=7

Or

(b) Give the statement of Rolle's theorem. Show that

$$\frac{\nu-\mu}{1+\nu^2}<\tan^{-1}\nu-\tan^{-1}\mu<\frac{\nu-\mu}{1+\mu^2},$$

if $0<\mu<\nu$ and deduce that

$$\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$$

2+5=7

6. (a) Define the operator E and show that $E=1+\Delta$. Represent the function $f(x)$ given by

$$f(x)=2x^4-12x^3+24x^2-30x+9$$

and its successive differences in factorial notation. What do you mean by interpolation? Write the statement of Newton's forward interpolation formula.

1+2+3+3=9

Or

(b) What do you mean by numerical integration? What are the basic conditions to apply Simpson's one-third rule? Solve $u_{x+1}-au_x=0;\ a\neq 1$. Evaluate $\sqrt{12}$ by applying Newton's formula.

2+2+2+3=9

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Choose the correct answer from the following : 1×8=8

(a) The set of natural numbers N is

(i) bounded above

(ii) bounded below

(iii) Both (i) and (ii)

(iv) None of the above

(b) If $S_{n+1} \geq S_n$, then the sequence $\{S_n\}$ is

(i) monotonic increasing

(ii) strictly increasing

(iii) monotonic decreasing

(iv) oscillatory

(c) The series Σu_n of positive terms is convergent or divergent as

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} > 1 \text{ or } < 1$$

Then the test is known as

(i) comparison test

(ii) Raabe's test

(iii) Cauchy's condensation test

(iv) D'Alembert's test

(d) The series

$$\Sigma u_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

is

(i) absolute convergent

(ii) conditional convergent

(iii) Both (i) and (ii)

(iv) None of the above

(e) The function $f(x) = |x| - 1;\ x \in \mathbb{R}$ is

(i) differentiable at $x = 0$

(ii) not differentiable at $x = 0$

(iii) continuous at $x = 1$

(iv) None of the above

(f) To which of the following, Rolle's theorem can be applied?

(i) $f(x) = \tan x$ in $[0, \pi]$

(ii) $f(x) = \cos\left(\frac{1}{x}\right)$ in $[-1, 1]$

(iii) $f(x) = x^2$ in $[2, 3]$

(iv) $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$

(g) The nth divided difference of a polynomial of nth degree is
- (i) always zero
- (ii) always equal to n
- (iii) always constant
- (iv) not defined

(h) Lagrange's formula is useful for
- (i) interpolation
- (ii) extrapolation
- (iii) inverse interpolation
- (iv) All of the above

2. Answer the following questions in brief : 2×8=16

(a) Define derived set with examples.

(b) Define limit superior of a bounded sequence.

(c) Define Raabe's test.

(d) Write the statement of L' Hospital rule.

(e) Write the properties of a continuous function.

(f) State the Taylor's theorem with the remainder in Cauchy's form.

(g) Under what situation is the Newton's method of backward difference of interpolation applicable?

(h) Write the statement of Weddle's rule.

3. Answer any *two* of the following questions :

(a) Define a bounded set and bounded sequence. If $\{a_n\}$ is a bounded sequence such that $a_n > 0$ for all $n \in \mathbb{N}$, then show that

$$\underline{\lim}\left\{\frac{1}{a_n}\right\} = \frac{1}{\overline{\lim}\, a_n}, \text{ if } \lim \overline{a_n} > 0$$ 2+5=7

(b) State the axioms of an ordered field. Show that the set of real numbers forms a complete ordered field. 2+5=7

(c) Define sequence and range of a sequence. Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}}\right]=1$$

1+1+5=7

4. Answer any *two* of the following questions :

(a) Define infinite series with examples. Under what condition a geometric series is convergent? Show that the series

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$$

is not convergent. 2+1+4=7

(b) Show that every absolutely convergent series is convergent. Test the convergence of the following series using Raabe's test : 3+4=7

$$\frac{\alpha}{\beta}+\frac{1+\alpha}{1+\beta}+\frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)}+\cdots$$

(c) State the Leibnitz's test for the convergence of alternating series. If the limit of

$$\frac{\sin 2x - a\sin x}{x^3}$$

as $x \to 0$ is finite, then find the value of a and the limit. 2+5=7

5. Answer any *two* of the following questions :

(a) Define uniform continuity of a function. Find the values of a and b, so that $f(x)$ may be differentiable at $x=1$, where

$$f(x)=\begin{cases} x^2+3x+a & ; \text{ if } x\le 1 \\ bx+2 & ; \text{ if } x>1 \end{cases}$$ 2+5=7

(b) Give the statement of Rolle's theorem. Show that

$$\frac{\nu-\mu}{1+\nu^2}<\tan^{-1}\nu-\tan^{-1}\mu<\frac{\nu-\mu}{1+\mu^2}$$

if $0<\mu<\nu$ and deduce that

$$\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$$ 2+5=7

(Continued)

(c) State Cauchy's mean value theorem. Expand $\cos x$ in powers of x in infinite series using Maclaurin's series expansion. 2+5=7

6. Answer any *two* of the following questions :

(a) Define the operators Δ and E. Show that if $h=1$

$$\Delta^2 \log x = \log\left[1-\frac{1}{(x+1)^2}\right]$$

Represent the function $f(x)$ given by

$$f(x)=2x^4-12x^3+24x^2-30x+9$$

and its successive differences in factorial notation. 1+3+3=7

(b) What do you mean by interpolation? When would you recommend the formula involving divided differences? Find the third divided difference with arguments 2, 4, 9, 10 of the function

$$f(x)=x^3-2x$$ 2+2+3=7

(c) What do you mean by numerical integration? Solve

$$u_{x+1}-au_x=0;\ a\ne 1$$

Evaluate $\sqrt{12}$ by applying Newton's formula. 2+2+3=7

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