Total No. of Printed Pages—12

3 SEM TDC STSH (CBCS) C 7 (N/O)

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(Nov/Dec)

STATISTICS

(Core)

Paper : C–7

(Mathematical Analysis)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct alternative out of the given ones : 1×6=6

(a) The set of all limit points of a set is called

(i) derived set *(ii)* open set

(iii) closed set *(iv)* bounded set

(b) The set of natural numbers N is
  (i) bounded above
  (ii) bounded below
  (iii) Both *(i)* and *(ii)*.
  (iv) None of the above

(c) The series $\sum u_n$ of positive terms is convergent or divergent as

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} > 1 \text{ or } < 1$$

Then the test is known as
  (i) comparison test
  (ii) Raabe's test
  (iii) D'Alembert's test
  (iv) Cauchy's condensation test

(d) The function $f(x) = |x| - 1;\ x \in \mathbb{R}$ is
  (i) differentiable at $x = 0$
  (ii) not differentiable at $x = 0$
  (iii) continuous at $x = 1$
  (iv) None of the above

(e) Δ and E operators are commutative w.r.t.
  (i) a variable
  (ii) a constant
  (iii) Both *(i)* and *(ii)*
  (iv) None of the above

(f) The n-th divided difference of a polynomial of n-th degree is
  (i) always zero
  (ii) always equal to n
  (iii) always constant
  (iv) not defined

2. Answer the following questions in brief : 2×6=12

(a) State the axioms of an ordered field.

(b) Define monotonic sequence.

(c) Define Raabe's test.

(d) State Rolle's theorem.

(e) State two properties of divided difference.

(f) Explain the use of numerical integration in Statistics.

3. *(a)* Define supremum and infimum of a set with examples. Show that a non-empty finite set cannot be an nbd of any of its points. 3+6=9

Or

(b) Show that every convergent sequence is bounded. Prove that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right] = 1$$

4+5=9

4. Answer any *two* of the following questions :

(a) Give a comparison test for positive term series $\sum u_n$ and $\sum v_n$. Test the convergence of the series

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\frac{7}{4.5.6}+\cdots$$

3+3=6

(b) Define Leibnitz's test for alternating series. Show that the series

$$\sum\frac{e^{-\lambda x}\lambda^x}{x!}, \quad \lambda>0$$

is convergent. 2+4=6

(c) Define Cauchy's n-th root test. If the limit of

$$\frac{\sin 2x - a\sin x}{x^3}$$

as $x \to 0$ is finite, find the value of a and the limit. 2+4=6

5. (a) Explain differentiability of a function with example. For what choice of a and b, the function

$$f(x)=\begin{cases} x^2; & x\le k \\ ax+b; & x>k \end{cases}$$

is differentiable at $x=k$? 3+4=7

Or

(b) Give Maclaurin's expansion for the function $f(x)$ with remainder term. Hence or otherwise show that

$$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}-\cdots(-1<x\le 1)$$

2+5=7

6. (a) (i) What do you mean by interpolation? State Lagrange's interpolation formula and discuss its merits and demerits. 1+3=4

(ii) Show that

$$e^x=\left(\frac{\Delta^2}{E}\right)e^x\cdot\frac{Ee^x}{\Delta^2 e^x}$$

When would you recommend the formulae involving divided differences? 3+2=5

Or

(b) (i) Define transcendental equations with examples. Evaluate $\sqrt{12}$ by applying Newton-Raphson method. 2+3=5

(ii) What are the basic conditions to apply Simpson's one-third rule? Solve $u_{x+1}-au_x=0;\ a\ne 1$. 2+2=4

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Choose the correct alternative out of the given ones : 1×8=8

   (a) The set of all limit points of a set is called

   (i) derived set *(ii)* open set

   (iii) closed set *(iv)* bounded set

   (b) The set of natural numbers N is

   (i) bounded above

   (ii) bounded below

   (iii) Both *(i)* and *(ii)*

   (iv) None of the above

   (c) The series $\sum u_n$ of positive terms is convergent or divergent as

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} > 1 \text{ or } < 1$$

   Then the test is known as

   (i) comparison test

   (ii) Raabe's test

   (iii) D'Alembert's test

   (iv) Cauchy's condensation test

   (d) If the series $\sum_{n=1}^{\infty} u_n$ converges, then

   (i) $\lim_{n\to\infty} u_n = 0$

   (ii) $\lim_{n\to\infty} u_n = 1$

   (iii) $\lim_{n\to\infty} u_n = -1$

   (iv) None of the above

   (e) The function $f(x) = |x| - 1;\ x \in \mathbb{R}$ is

   (i) differentiable at $x = 0$

   (ii) not differentiable at $x = 0$

   (iii) continuous at $x = 1$

   (iv) None of the above

   (f) Lagrange's form of remainder after n terms in Taylor's development of the function e^x in a finite form in the interval $[a, a+h]$ is

   (i) $\frac{h^n}{n!} e^{a+\theta h}$

   (ii) $\frac{h^n}{n!} e^{\theta h}$

   (iii) $\frac{h^n (1-\theta)}{n!} e^{a+\theta h}$

   (iv) $\frac{h^n (1+\theta)^n}{n!} e^{a+\theta h}$

(g) Δ and E operators are commutative w.r.t.

(i) a variable

(ii) a constant

(iii) Both *(i)* and *(ii)*

(iv) None of the above

(h) The n-th divided difference of a polynomial of n-th degree is

(i) always zero

(ii) always equal to n

(iii) always constant

(iv) not defined

2. Answer the following questions in brief : 2×8=16

(a) State the axioms of an ordered field.

(b) Define monotonic sequence.

(c) Define Raabe's test.

(d) Define absolute convergence and conditional convergence of a series.

(e) Define uniform continuity of a function.

(f) State Rolle's theorem.

(g) State two properties of divided difference.

(h) Explain the use of numerical integration in Statistics.

3. Answer any *two* of the following questions :

(a) Define supremum and infimum of a set with examples. Show that a non-empty finite set cannot be an nbd of any of its points. 2+5=7

(b) Show that every convergent sequence is bounded. Prove that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}}\right]=1$$

3+4=7

(c) State Cauchy's general principle of convergence. If $\{a_n\}$ is a bounded sequence such that $a_n > 0$ for all $n \in N$, then show that

$$\underline{\lim}\left\{\frac{1}{a_n}\right\}=\frac{1}{\overline{\lim}\, a_n}, \text{ if } \overline{\lim}\, a_n > 0$$

2+5=7

4. Answer any *two* of the following questions :

(a) Give a comparison test for positive term series $\sum u_n$ and $\sum v_n$. Test the convergence of the series

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\frac{7}{4.5.6}+\cdots$$

2+5=7

(b) Define Leibnitz's test for alternating series. Show that the series

$$\sum\frac{e^{-\lambda x}\lambda^x}{x!},\quad \lambda>0$$

is convergent.

2+5=7

(c) Define Cauchy's n-th root test. If the limit of

$$\frac{\sin 2x-a\sin x}{x^3}$$

as $x\to 0$ is finite, find the value of a and the limit.

2+5=7

5. Answer any *two* of the following questions :

(a) Explain with example the differentiability of function. For what choice of a and b, the function

$$f(x)=\begin{cases} x^2; & x\le k \\ ax+b; & x>k \end{cases}$$

is differentiable at $x=k$?

3+4=7

(Continued)

(b) Give Maclaurin's expansion for the function $f(x)$ with remainder term. Hence or otherwise show that

$$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}-\cdots(-1<x\le 1)$$

2+5=7

(c) State Lagrange's mean value theorem and show that

$$\frac{v-u}{1+v^2}<\tan^{-1}v-\tan^{-1}u<\frac{v-u}{1+u^2}$$

2+5=7

6. Answer any *two* of the following questions :

(a) Define the operators Δ and E. Show that

$$e^x=\left(\frac{\Delta^2}{E}\right)e^x\cdot\frac{Ee^x}{\Delta^2 e^x}$$

represents the function $f(x)$ given by

$$f(x)=2x^4-12x^3+24x^2-30x+9$$

and its successive differences in factorial notation.

1+3+3=7

(Turn Over)

(b) When would you recommend the formula involving divided differences? Find the third divided difference with arguments 2, 4, 9, 10 of the function $f(x)=x^3-2x$. What are the advantages of Lagrange's interpolation formula?

2+3+2=7

(c) What are the basic conditions to apply Simpson's three-eighths rule? Solve

$$u_{x+1}-au_x=0;\ a\neq 1$$

Evaluate $\sqrt{12}$ by applying Newton-Raphson method. 2+2+3=7

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