

Total No. of Printed Pages—11

3 SEM TDC STSH (CBCS) C 5 (N/O)

2025

(Nov/Dec)

STATISTICS

(Core)

Paper : C-5

(Sampling Distribution)

*The figures in the margin indicate full marks
for the questions*

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following alternatives : 1×6=6

(a) If X is a continuous random variable with mean μ and variance σ^2 , then for any positive number k

$$P\{|x - \mu| \geq k\sigma\} \leq \frac{1}{k^2}$$

(2)

is known as

- (i) Chebychev's inequality
 - (ii) weak law of large numbers
 - (iii) strong law of large numbers
 - (iv) None of the above
- (b) The SE of mean of a random sample of size n from a population with variance 4 is
- (i) 2
 - (ii) $\frac{2}{n}$
 - (iii) $\frac{2}{\sqrt{n}}$
 - (iv) $\frac{n}{2}$
- (c) Level of significance is the probability of
- (i) type I error
 - (ii) type II error
 - (iii) not committing error
 - (iv) Any of the above

(3)

(d) The range of χ^2 -variate is

- (i) $-\infty$ to ∞
 - (ii) 0 to ∞
 - (iii) 0 to 1
 - (iv) $-\infty$ to 0
- (e) The mean of Chi-square distribution with 12 d.f. is
- (i) 12
 - (ii) 11
 - (iii) $\sqrt{12}$
 - (iv) 24
- (f) The variance of Student's t -distribution is
- (i) n
 - (ii) $2n$
 - (iii) $\frac{v}{v-2}$ where v = degrees of freedom
 - (iv) None of the above

(4)

2. Answer the following questions in brief :

$$2 \times 6 = 12$$

- (a) Define convergence in probability.
- (b) What do you mean by standard error of a statistic?
- (c) Distinguish between parameter and statistic.
- (d) State additive property of χ^2 -variate.
- (e) State the assumptions of Student's t-test.
- (f) Define F-statistic.

3. (a) Distinguish between weak law of large numbers and strong law of large numbers. Give the statement of De-Moivre-Laplace central limit theorem. A coin is tossed 200 times. Find the approximate probability that the number of heads obtained is between 80 and 120.

$$2 + 2 + 5 = 9$$

Or

- (b) Obtain the cumulative distribution function of a single-order statistic and find the probability density function.

$$4 + 5 = 9$$

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(Continued)

(5)

4. (a) Obtain the sampling distribution of mean of a random sample drawn from a normal population with mean μ and variance σ^2 . Define null and alternative hypothesis with examples.

$$5 + 3 = 8$$

Or

- (b) Illustrate with examples two types of error occurred in testing of hypothesis. Describe how you would test the difference of two means in large samples.

$$4 + 4 = 8$$

5. (a) Prove that the sum of independent χ^2 -variate is also a χ^2 -variate. How do you test the independence of attribute with it?

$$5 + 5 = 10$$

Or

- (b) Obtain moment generating function and cumulant generating function of χ^2 -distribution. Obtain its first four cumulants and hence its mean and variance.

$$5 + 5 = 10$$

26P/184

(Turn Over)

(6)

6. (a) Define Student's t -statistic. Mention some applications of Student's t -test in tests of significance. Explain the t -test for testing the significance of the difference between two sample means.

1+4+5=10

Or

- (b) Write down the probability density function of F -distribution. Obtain the mode of F -distribution with (n_1, n_2) d.f. Establish the relationship between t and F distributions.

1+4+5=10

(7)

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

1. Choose the correct answer from the following alternatives : 1×5=5

- (a) If X is a continuous random variable with mean μ and variance σ^2 , then for any positive number k

$$P\{|x - \mu| \geq k\sigma\} \leq \frac{1}{k^2}$$

is known as

- (i) Chebychev's inequality
 - (ii) weak law of large numbers
 - (iii) strong law of large numbers
 - (iv) None of the above
- (b) The SE of mean of a random sample of size n from a population with variance 4 is
- (i) 2
 - (ii) $\frac{2}{n}$
 - (iii) $\frac{2}{\sqrt{n}}$
 - (iv) $\frac{n}{2}$

(c) Level of significance is the probability of

- (i) type I error
- (ii) type II error
- (iii) not committing error
- (iv) Any of the above

(d) The mean Chi-square distribution with 12 d.f. is

- (i) 12
- (ii) 11
- (iii) $\sqrt{12}$
- (iv) 24

(e) The variance of Student's t -distribution is

- (i) n
- (ii) $2n$
- (iii) $\frac{\nu}{\nu-2}$ where ν = degrees of freedom
- (iv) None of the above

2. Answer the following questions in brief :

2×5=10

- (a) Define convergence in probability.
- (b) What do you mean by standard error of a statistic?
- (c) State additive property of χ^2 -variate.
- (d) State the assumptions of Student's t -test.
- (e) Define F -statistic.

3. (a) Distinguish between weak law of large numbers and strong law of large numbers. Give the statement of De-Moivre-Laplace central limit theorem. A coin is tossed 200 times. Find the approximate probability that the number of heads obtained is between 80 and 120.

2+2+5=9

Or

- (b) Obtain the cumulative distribution function of a single-order statistic and find the probability density function.

4+5=9

(10)

4. (a) Obtain the sampling distribution of mean of a random sample drawn from a normal population with mean μ and variance σ^2 . Define null and alternative hypothesis with examples. 5+3=8

Or

- (b) Illustrate with examples two types of error occurred in testing of hypothesis. Describe how you would test the difference of two means in large samples. 4+4=8

5. (a) Prove that the sum of independent χ^2 -variate is also a χ^2 -variate. How do you test the independence of attribute with it? 5+4=9

Or

- (b) Obtain moment generating function and cumulant generating function of χ^2 -distribution. Obtain its first four cumulants and hence its mean and variance. 5+4=9

(11)

6. (a) Define Student's t -statistic. Mention some applications of Student's t -test in tests of significance. Explain the t -test for testing the significance of the difference between two sample means. 1+4+4=9

Or

- (b) Write down the probability density function of F -distribution. Obtain the mode of F -distribution with (n_1, n_2) d.f. Establish the relationship between t and F distributions. 1+4+4=9

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3 SEM TDC STSH (CBCS) C 6 (N/O)

2 0 2 5

(Nov/Dec)

STATISTICS

(Core)

Paper : C-6

(Survey Sampling and Indian Official Statistics)

*The figures in the margin indicate full marks
for the questions*

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer of each question
from given alternatives : 1×6=6

(a) The number of possible samples of
size n from a population of N units with
replacement is

(i) $N!$

(ii) $\binom{N}{n}$

(iii) N^n

(iv) n^2

(2)

- (b) Sampling fraction is considered to be negligible, if it is
- (i) $< 10\%$
 - (ii) $\leq 5\%$
 - (iii) $> 5\%$
 - (iv) $> 10\%$
- (c) Regarding the number of strata, which statement is true?
- (i) Lesser the number of strata, better it is.
 - (ii) More the number of strata, worse it is.
 - (iii) More the number of strata, better it is.
 - (iv) None of the above
- (d) Which of the following advantages of systematic sampling do you approve?
- (i) Easy selection of sample
 - (ii) Economical
 - (iii) Spread of sample over the whole population
 - (iv) All of the above

(3)

- (e) Under proportional allocation, the size of the sample from each stratum depends on
- (i) total sample size
 - (ii) size of the stratum
 - (iii) population size
 - (iv) All of the above
- (f) In which of the following situations, cluster sampling is appropriate?
- (i) When the units are situated far apart
 - (ii) When the sampling frame is not available
 - (iii) When all the elementary units are not easily identifiable
 - (iv) All of the above

2. Answer the following questions in brief :

$2 \times 7 = 14$

- (a) What are non-sampling errors?
- (b) What do you mean by sampling frame?
- (c) What is the key feature of simple random sampling?

(4)

- (d) What is proportional allocation in stratified random sampling?
- (e) Explain circular systematic sampling.
- (f) In what way linear regression estimate differs from ratio estimate?
- (g) What is the role of NSO in India? Explain briefly.

3. (a) Discuss briefly the main steps involved in a sample survey. Enumerate the advantages of a sample survey over complete enumeration. 5+5=10

Or

- (b) What are the different sources of errors in sample survey? Describe the measures to be taken in controlling these errors. Elucidate judgement sampling and probability sampling. 3+4+3=10

4. (a) (i) What do you mean by simple random sampling with replacement and without replacement from a finite population?
- (ii) Show that in SRSWOR, the sample mean is an unbiased estimator of the population mean, i.e., $E(\bar{Y}_n) = \bar{Y}_N$.

26P/185

(Continued)

(5)

- (iii) Show that in SRSWOR, the variance of the sample mean is given by

$$\text{var}(\bar{Y}_n) = \frac{S^2}{n} \frac{N-n}{N} \quad 4+3+3=10$$

Or

- (b) (i) What do you mean by simple random sampling of attributes?

- (ii) In a population of N units, the number of units possessing a certain characteristic is A , and in a simple random sample of size n from it, the number of units possessing that characteristic is a .

If $P = \frac{A}{N}$, $p = \frac{a}{n}$, $Q = 1 - P$ and

$q = 1 - p$, then show that p is an unbiased estimator of population proportion P and $\text{var}(p) = \frac{N-n}{N-1} \frac{PQ}{n}$.

3+3+4=10

Or

- (c) Describe the procedure of stratified random sampling. Under what conditions is stratified random sampling preferred over simple random sampling and why? Prove that $\text{var}(\bar{Y}_{st})$ is minimum for fixed total size of the sample (n) if $n_i \propto N_i S_i$.

2+3+5=10

26P/185

(Turn Over)

5. (a) What do you mean by ratio estimator? Obtain the bias of the ratio estimator $\hat{R}$ in simple random sampling. Show that the fixed approximation to the relative bias of the ratio estimator in simple random sampling without replacement is given by

$$\frac{B(\hat{R})}{\hat{R}} \cong \frac{(1-f)}{n} (C_x^2 - \rho C_x C_y)$$

where $C_x = \frac{S_x}{\bar{X}}$ and $C_y = \frac{S_y}{\bar{y}}$ are the coefficients of variation of x and y respectively.
2+2+5=9

Or

- (b) What is cluster sampling? In what situation is the cluster sampling preferred? Show that in simple random sampling without replacement of n clusters each containing M elements from a population of N clusters, the sample mean $\bar{y}_n$ is an unbiased estimator of $\bar{y}$ and its variance is given by

$$\text{var}(\bar{y}_n) \cong \left(\frac{1-f}{n} \right) S_M^2 \{1 + (M-1)\rho\}$$

where ρ is the intraclass correlation coefficient.
2+2+5=9

6. (a) Discuss the role and importance of official statistics in India, highlighting their use in economic planning and policy-making. Mention two examples of official statistics collected in India. 6

Or

- (b) Describe in brief the functions of the Central Statistical Organization and name at least five of its publications. 6

(8)

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

1. Choose the correct answer of each question from given alternatives : $1 \times 5 = 5$

(a) The number of possible samples of size n from a population of N units with replacement is

(i) $N!$

(ii) $\binom{N}{n}$

(iii) N^n

(iv) n^2

(b) Sampling fraction is considered to be negligible, if it is

(i) $\leq 10\%$

(ii) $\leq 5\%$

(iii) $> 5\%$

(iv) $> 10\%$

(c) Regarding the number of strata, which statement is true?

(i) Lesser the number of strata, better it is.

(ii) More the number of strata, worse it is.

(iii) More the number of strata, better it is.

(iv) None of the above

26P/185

(Continued)

(9)

(d) Which of the following advantages of systematic sampling do you approve?

(i) Easy selection of sample

(ii) Economical

(iii) Spread of sample over the whole population

(iv) All of the above

(e) Under proportional allocation, the size of the sample from each stratum depends on

(i) total sample size

(ii) size of the stratum

(iii) population size

(iv) All of the above

2. Answer the following questions in brief :

$2 \times 5 = 10$

(a) What are non-sampling errors?

(b) What do you mean by sampling frame?

(c) What is the key feature of simple random sampling?

(d) Explain circular systematic sampling.

(e) What is the role of NSO in India? Explain briefly.

26P/185

(Turn Over)

3. (a) Discuss briefly the main steps involved in a sample survey. Enumerate the advantages of a sample survey over complete enumeration. 5+5=10

Or

- (b) What are the different sources of errors in sample survey? Describe the measures to be taken in controlling these errors. Elucidate judgement sampling and probability sampling. 3+4+3=10

4. (a) (i) What do you mean by simple random sampling with replacement and without replacement from a finite population?
- (ii) Show that in SRSWOR, the sample mean is an unbiased estimator of the population mean, i.e., $E(\bar{y}_n) = \bar{Y}_N$.
- (iii) Show that in SRSWOR, the variance of the sample mean is given by

$$\text{var}(\bar{y}_n) = \frac{S^2}{n} \frac{N-n}{N} \quad 4+3+3=10$$

Or

- (b) (i) What do you mean by simple random sampling of attributes?
- (ii) In a population of N units, the number of units possessing a certain characteristic is A , and in a simple random sample of size n from it, the number of units possessing that characteristic is a .

(Continued)

If $P = \frac{A}{N}$, $p = \frac{a}{n}$, $Q = 1 - P$ and $q = 1 - p$, then show that p is an unbiased estimator of population proportion P and $\text{var}(p) = \frac{N-n}{N-1} \frac{PQ}{n}$. 3+3+4=10

Or

- (c) Describe the procedure of stratified random sampling. Under what conditions is stratified random sampling preferred over simple random sampling and why? Prove that $\text{var}(\bar{y}_{st})$ is minimum for fixed total size of the sample (n) if $n_i \propto N_i S_i$. 2+3+5=10

5. (a) What do you mean by ratio estimator? Obtain the bias of the ratio estimator $\hat{R}$ in simple random sampling. Show that the fixed approximation to the relative bias of the ratio estimator in simple random sampling without replacement is given by

$$\frac{B(\hat{R})}{R} = \frac{(1-f)}{n} (C_x^2 - \rho C_x C_y)$$

where $C_x = \frac{S_x}{\bar{X}}$ and $C_y = \frac{S_y}{\bar{Y}}$ are the coefficients of variation of x and y respectively. 2+2+5=9

(Turn Over)

Or

- (b) What is cluster sampling? In what situation is the cluster sampling preferred? Show that in simple random sampling without replacement of n clusters each containing M elements from a population of N clusters, the sample mean $\bar{y}_n$ is an unbiased estimator of $\bar{y}$ and its variance is given by

$$\text{var}(\bar{y}_n) = \left(\frac{1-f}{n} \right) S_M^2 \{1 + (M-1)\rho\}$$

where ρ is the intracluster correlation coefficient.

2+2+5=9

6. (a) Discuss the role and importance of official statistics in India, highlighting their use in economic planning and policy-making. Mention two examples of official statistics collected in India. 6

Or

- (b) Describe in brief the functions of the Central Statistical Organization and name at least five of its publications. 6

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Total No. of Printed Pages—11

3 SEM TDC STSH (CBCS) C 7 (N/O)

2025

(Nov/Dec)

STATISTICS

(Core)

Paper : C-7

(Mathematical Analysis)

*The figures in the margin indicate full marks
for the questions*

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer : 1×5=5

(a) The greatest lower bound of the set of
all positive even integers is

(i) 0

(ii) 1

(iii) 2

(iv) None of the above

(2)

- (b) The infimum and supremum of the set $\{1, 3, 5, 7, 9\}$ are
- (i) 1, 5
 - (ii) 3, 7
 - (iii) 1, 9
 - (iv) All of the above
- (c) Infinite series $\sum \frac{1}{n^p}$ is convergent if
- (i) $p < 1$
 - (ii) $p > 1$
 - (iii) $p = 1$
 - (iv) $p \leq 1$
- (d) A bounded sequence $\{a_n\}$ converges to a real number 'a' if and only if
- (i) $\lim a_n = a$
 - (ii) $\overline{\lim} a_n = a$
 - (iii) $\lim a_n = \overline{\lim} a_n = a$
 - (iv) All of the above
- (e) The function $f(x) = |x+2|$ is not differentiable at a point
- (i) $x = 2$
 - (ii) $x = -2$
 - (iii) $x = -1$
 - (iv) $x = 1$

(3)

2. Answer the following questions : 2×5=10
- (a) Define bounded and unbounded set.
 - (b) What is neighbourhood of a set?
 - (c) Define a sequence of real numbers with example.
 - (d) Define convergence and divergence of a series.
 - (e) Show that the necessary and sufficient condition for convergence of an infinite series $\sum u_n$ is that

$$\sum_{n \rightarrow \infty} u_n = 0$$

3. Answer any two of the following questions :

- (a) Define derived set, open set and closed set. Prove that the union of two closed sets is a closed set. 2+3=5
- (b) Prove that the Cartesian product of two countable sets is countable. Prove that the set R of real numbers is uncountable. 2+3=5
- (c) What is a monotonic sequence? Show that

$$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}, \forall n \in N$$

is convergent.

2+3=5

(4)

4. Answer any *two* of the following questions :

- (a) What is Cauchy's root test? Test the convergence of the series whose general term is

$$\left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{1/2}} \quad 2+3=5$$

- (b) Define Gauss's test. Test for convergence of the series

$$\sum \frac{1^2 \cdot 3^2 \cdots (2n-1)^2}{2^2 \cdot 4^2 \cdots (2n)^2} \cdot x^{n-1}, \quad x > 0 \quad 2+3=5$$

- (c) What is Leibnitz's test for the convergence of alternating series? Prove that the series

$$\sum \frac{(-1)^{n+1}}{\sqrt{n}}$$

is conditionally convergent. $2+3=5$

5. Answer any *two* of the following questions :

- (a) Define differentiability of a function in an interval $[a, b]$. Define uniform continuity. Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $]0, 1[$.

$2+3=5$

(5)

- (b) State Rolle's theorem. Verify whether the function $f(x) = \sin x$ in $[0, \pi]$ satisfies the conditions of Rolle's theorem and hence find C as prescribed by the theorem. $2+3=5$

- (c) State Taylor's theorem with Cauchy's form of remainder and obtain Maclaurin's series expansion of $\cos x$. $2+3=5$

6. Answer any *two* of the following questions :

- (a) What do you mean by interpolation? State the assumptions of interpolation. Prove Newton's formula for forward interpolation. $2+3=5$

- (b) Deduce Lagrange's formula for interpolation. Distinguish between Newton's divided difference formula and Lagrange's interpolation formula. $2+3=5$

- (c) What is transcendental equation? By using Newton-Raphson method, find the root of $x^4 - x - 10 = 0$ which is nearer to $x = 2$, correct to three decimal places. $2+3=5$

(6)

(Old Course)

Full Marks : 80

Pass Marks : 24

Time : 3 hours

1. Choose the correct answer : $1 \times 8 = 8$

(a) The greatest lower bound of the set of all positive even integers is

(i) 0

(ii) 1

(iii) 2

(iv) None of the above

(b) The infimum and supremum of the set $\{1, 3, 5, 7, 9\}$ are

(i) 1, 5

(ii) 3, 7

(iii) 1, 9

(iv) All of the above

(c) Infinite series $\sum \frac{1}{n^p}$ is convergent if

(i) $p < 1$

(ii) $p > 1$

(iii) $p = 1$

(iv) $p \leq 1$

(7)

(d) A bounded sequence $\{a_n\}$ converges to a real number 'a' if and only if

(i) $\lim a_n = a$

(ii) $\overline{\lim} a_n = a$

(iii) $\lim a_n = \overline{\lim} a_n = a$

(iv) All of the above

(e) The function $f(x) = |x+2|$ is not differentiable at a point

(i) $x = 2$

(ii) $x = -2$

(iii) $x = -1$

(iv) $x = 1$

(f) Which one of the following is not a condition of Rolle's theorem to be defined for $f(x)$ on $[a, b]$?

(i) $f'(x) > 0 \forall x \in [a, b]$

(ii) $f(x)$ is continuous on $[a, b]$

(iii) $f(x)$ is derivable on $[a, b]$

(iv) $f(a) = f(b)$

- (g) If $f(x)$ be a polynomial of n -th degree, then

(i) $\Delta^n f(x) = 0$

(ii) $\Delta^{n-1} f(x) = \text{constant}$

(iii) $\Delta^{n+1} f(x) = 0$

(iv) $\Delta^{n+1} f(x) = \text{constant}$

- (h) Simpson's one-third rule is called

(i) straight line formula

(ii) hyperbolic formula

(iii) parabolic formula

(iv) None of the above

2. Answer the following questions : $2 \times 8 = 16$

- (a) Define bounded and unbounded set.
 (b) What is neighbourhood of a set?
 (c) Define a sequence of real numbers with example.
 (d) Define convergence and divergence of a series.

- (e) Show that the necessary and sufficient condition for convergence of an infinite series $\sum u_n$ is that

$$\sum_{n \rightarrow \infty} u_n = 0$$

- (f) State Taylor's theorem.

- (g) Evaluate :

$$\frac{\Delta}{E} x^3$$

- (h) State the assumptions on which interpolation is based.

3. Answer any two of the following questions :

- (a) Define derived set, open set and closed set. Prove that the union of two closed sets is a closed set. $3+4=7$
 (b) Prove that the Cartesian product of two countable sets is countable. Prove that the set R of real numbers is uncountable. $4+3=7$
 (c) What is a monotonic sequence? Show that

$$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}, \forall n \in N$$

is convergent.

$$2+5=7$$

4. Answer any *two* of the following questions :

- (a) What is Cauchy's root test? Test the convergence of the series whose general term is

$$\left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{1/2}} \quad 2+5=7$$

- (b) Define Gauss's test. Test for convergence of the series

$$\sum \frac{1^2 \cdot 3^2 \cdot \dots \cdot (2n-1)^2}{2^2 \cdot 4^2 \cdot \dots \cdot (2n)^2} \cdot x^{n-1}, \quad x > 0 \quad 2+5=7$$

- (c) What is Leibnitz's test for the convergence of alternating series? Prove that the series

$$\sum \frac{(-1)^{n+1}}{\sqrt{n}}$$

is conditionally convergent. $2+5=7$

5. Answer any *two* of the following questions :

- (a) Define differentiability of a function in an interval $[a, b]$. Define uniform continuity. Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $]0, 1[$.

$2+2+3=7$

- (b) State Rolle's theorem. Verify whether the function $f(x) = \sin x$ in $[0, \pi]$ satisfies the conditions of Rolle's theorem and hence find C as prescribed by the theorem. $3+4=7$

- (c) State Taylor's theorem with Cauchy's form of remainder and obtain Maclaurin's series expansion of $\cos x$. $2+5=7$

6. Answer any *two* of the following questions :

- (a) What do you mean by interpolation? State the assumptions of interpolation. Prove Newton's formula for forward interpolation. $1+2+4=7$

- (b) Deduce Lagrange's formula for interpolation. Distinguish between Newton's divided difference formula and Lagrange's interpolation formula. $5+2=7$

- (c) What is transcendental equation? By using Newton-Raphson method, find the root of $x^4 - x - 10 = 0$ which is nearer to $x = 2$; correct to three decimal places. $2+5=7$
