

Total No. of Printed Pages—10

3 SEM TDC STSH (CBCS) C 5 (N/O)

2 0 2 4

(Nov/Dec)

STATISTICS

(Core)

Paper : C-5

(Sampling Distributions)

*The figures in the margin indicate full marks
for the questions*

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following alternatives : 1×6=6

(a) The mean of chi-square distribution with 12 d.f. is

(i) 12

(ii) 11

(iii) $\sqrt{12}$

(iv) 24

(2)

(b) The area of the critical region depends on the

- (i) size of type I error
- (ii) size of type II error
- (iii) number of observations
- (iv) value of the statistic

(c) The shape of chi-square distribution curve for χ^2 with d.f. 1 is

- (i) a parabola
- (ii) a hyperbola
- (iii) an inverted J-shaped curve
- (iv) a bell-shaped curve

(d) The mean of the F -distribution with degrees of freedom n_1 and n_2 for $n_2 > 2$ is

(i) $\frac{n_2}{n_1 - 2}$

(ii) $\frac{n_1}{n_2 - 2}$

(iii) $\frac{n_1}{n_1 - 2}$

(iv) $\frac{n_2}{n_2 - 2}$

(e) The degrees of freedom for Student's t -distribution based on a random sample of size n is

(i) $n - 1$

(ii) n

(iii) $n - 2$

(iv) $\frac{n - 1}{2}$

P25/360

(Continued)

(3)

(f) What is the condition for the validity of χ^2 -test?

- (i) The sample observations should be independent
- (ii) Constraints on the cell frequencies should be linear
- (iii) Cell frequency should be greater than 5
- (iv) All of the above

2. Answer the following questions : 2×6=12

- (a) What do you understand by null hypothesis and alternative hypothesis? Explain with examples.
- (b) Mention the null hypothesis to be tested and also the test statistics used in large sample test for single mean.
- (c) What is chi-square variate?
- (d) State additive property of χ^2 -variates.
- (e) State the assumptions for Student's t -test.
- (f) Describe briefly one application of F -distribution.

P25/360

(Turn Over)

3. (a) State and prove weak law of large numbers. Examine whether the weak law of large numbers holds for the sequence $\{X_k\}$ of independent random variables defined as follows :

$$P(X_k = \pm 2^k) = 2^{-(2k+1)}$$

$$P(X_k = 0) = 1 - 2^{-2k}$$

$$1+4+4=9$$

Or

- (b) Obtain the cumulative distribution function of a single-order statistic and find the probability density function of it. 9
4. (a) What are the required conditions for the validity of χ^2 -test of goodness of fit between theory and experiment? 4
- (b) Find the mode of χ^2 -distribution with n degrees of freedom. 4
- Or
- (c) If χ_1^2 and χ_2^2 are two independent χ^2 -variates with n_1 and n_2 degrees of freedom respectively, then show that $\frac{\chi_1^2}{\chi_2^2}$ is a $\beta_2\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$. 4
- (d) Show that χ^2 -distribution approaches to normality when $n \rightarrow \infty$. 4

5. (a) Define Student's t -distribution and derive its probability distribution. $3+7=10$

Or

- (b) Define F -statistic. Write down its probability density function. Describe the F -test for testing the significance of the equality of two population variances. $2+2+6=10$

6. Describe, in detail, how you will test the—

- (a) difference of proportions for large sample;
- (b) difference of standard deviations for large sample. $5+5=10$

Or

Describe in detail, how you will test the—

- (c) difference of means for large sample;
- (d) difference of standard deviations for large sample. $5+5=10$

(6)

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

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(iv) 24

(b) The area of the critical region depends on the

(i) size of type I error

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(iii) number of observations

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(c) The shape of chi-square distribution curve for χ^2 with d.f. 1 is

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(ii) an inverted J-shaped curve

(iii) a hyperbola

(iv) a bell-shaped curve

P25/360

(Continued)

(7)

- (d) The mean of the F-distribution with degrees of freedom n_1 and n_2 for $n_2 > 2$ is

(i) $\frac{n_2}{n_1 - 2}$

(ii) $\frac{n_1}{n_2 - 2}$

(iii) $\frac{n_1}{n_1 - 2}$

(iv) $\frac{n_2}{n_2 - 2}$

- (e) The degrees of freedom for Student's t-distribution based on a random sample of size n is

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P25/360

(Turn Over)

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$$1 + 4 + 4 = 9$$

(Continued)

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(b) Find the mode of χ^2 -distribution with n degrees of freedom. 4

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(d) Show that χ^2 -distribution approaches to normality when $n \rightarrow \infty$. 4

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(Turn Over)

Or

- (b) Define F -statistic. Write down its probability density function. Describe the F -test for testing the significance of the equality of two population variances. 2+2+6=10

6. Describe, in detail, how you will test the—

- (a) difference of proportions for large sample; 5

Or

- (b) difference of standard deviations for large sample. 5

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Total No. of Printed Pages—11

3 SEM TDC STSH (CBCS) C 6 (N/O)

2 0 2 4

(Nov/Dec)

STATISTICS

(Core)

Paper : C-6

(Survey Sampling and Indian Official Statistics)

*The figures in the margin indicate full marks
for the questions*

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer of each question
from given alternatives : $1 \times 6 = 6$

(a) The number of all possible samples of
size 2 from a population of size 6
without replacement is

(i) 12

(ii) 15

(iii) 30

(iv) None of the above

(2)

(b) The selection procedure of a sample having no involvement of probability is known as

- (i) purposive sampling
- (ii) subjective sampling
- (iii) judgement sampling
- (iv) All of the above

(c) In SRSWR, the variance of the sample mean is

- (i) $\frac{\sigma^2}{n} \frac{N-n}{N}$
- (ii) $\frac{S^2}{n} \frac{N-n}{N}$
- (iii) $\frac{\sigma^2}{n}$
- (iv) None of the above

(d) Which of the following statements is correct?

- (i) Systematic sample is superior than stratified random sample.
- (ii) Simple random sample is inferior than systematic sample.
- (iii) Stratified random sample is better than systematic sample.
- (iv) None of the above

(3)

(e) In which of the following situations, cluster sampling is appropriate?

- (i) When the units are situated far apart
- (ii) When sampling frame is not available
- (iii) When all the elementary units are not easily identifiable
- (iv) All of the above

(f) In ratio method of estimation

- (i) the relation between y_i and x_i is a straight line through the origin
- (ii) the relation between y_i and x_i is a straight line through any point
- (iii) the relation between y_i and x_i is non-linear
- (iv) None of the above

2. Answer the following : 2×6=12

- (a) What do you mean by sampling frame?
- (b) Describe the technique of drawing a stratified random sample.

- (c) State the circumstances when systematic sampling is optimum.
- (d) What are the main differences between cluster sampling and stratified random sampling?
- (e) State some of the importances of official statistics for public policy making.
- (f) In what way, linear regression estimate differs from ratio estimate?
3. (a) What are the different sources of errors in a sample survey? Describe briefly how these errors can be controlled.

5+5=10

Or

- (b) What do you understand by simple random sampling? Show that the probability of selecting a specified unit of the population at any given draw is equal to the probability of its being selected at the first draw in simple random sampling without replacement. Draw all possible samples of size two by using SRSWOR technique from the population [7, 12, 15, 20] and verify that the sample mean is an unbiased estimate of the population mean. 6+4=10

4. (a) What is stratified random sampling? Describe the methods used to fix the number of units to be selected from each stratum. Obtain $V(\bar{y}_{st})$ and also obtain the result for the particular cases when—
- (i) n_h / N_h is negligible;
- (ii) $n_h = \frac{nN_h}{N}$. 2+3+3+1+1=10

Or

- (b) Explain the method of systematic sampling. If the population consists of a linear trend, then prove that

$$\text{var}(\bar{y}_{st}) \leq \text{var}(\bar{y}_{sys}) \leq \text{var}(\bar{y}_n) \quad 3+7=10$$

5. (a) What do you mean by ratio estimate? With usual notations, derive the approximate variances of the ratio estimates $\hat{R}$ and $\hat{Y}_R$. 2+2+6=10

Or

- (b) What is cluster sampling? In what situation, the cluster sampling be preferred? Show that in simple random sampling without replacement of n clusters each containing M elements from a population of N clusters, the

(6)

sample mean $\bar{y}_n$ is an unbiased estimator of $\bar{y}$ and its variance is given by

$$\text{var}(\bar{y}_n) = \left(\frac{1-f}{n} \right) S_M^2 \{1 + (M-1)\rho\}$$

where ρ is the intraclass correlation coefficient.
2+2+6=10

6. (a) Write a brief report on the official statistical system in India. 7

Or

- (b) Write a note on the National Sample Survey Organization. Write about two special features of National Sample Survey Organization.

(7)

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

1. Choose the correct answer of each question from given alternatives : 1×5=5

- (a) The number of all possible samples of size 2 from a population of size 6 without replacement is

(i) 12

(ii) 15

(iii) 30

(iv) None of the above

- (b) The selection procedure of a sample having no involvement of probability is known as

(i) purposive sampling

(ii) subjective sampling

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(iv) All of the above

- (c) In SRSWR, the variance of the sample mean is

(i) $\frac{\sigma^2}{n} \frac{N-n}{N}$

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2×5=10

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