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4 SEM TDC STSH (CBCS) C 8 (N/O)

2025

(May/June)

STATISTICS

(Core)

Paper : C–8

(Statistical Inference)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Select the correct one out of the given alternatives : 1×6=6

(a) If the expected value of an estimator is not equal to its parametric function $r(\theta)$, it is said to be a/an

(i) unbiased estimator

(ii) biased estimator

(iii) consistent estimator

(iv) None of the above

(b) Factorisation theorem for sufficiency is known as

(i) Rao-Blackwell theorem

(ii) Cramer-Rao theorem

(iii) Fisher-Neyman theorem

(iv) None of the above

(c) The maximum likelihood estimators which are obtained by maximizing the function of joint density of random variables are generally

(i) unbiased and consistent

(ii) unbiased and inconsistent

(iii) consistent and invariant

(iv) invariant and unbiased

(d) The area of the critical region depends on the

(i) size of type I error

(ii) size of type II error

(iii) value of the statistics

(iv) number of observations

(e) The range of likelihood ratio statistics (λ) is

(i) 0 to 1

(ii) $-\infty$ to ∞

(iii) -1 to 1

(iv) 0 to ∞

(f) In sequential probability ratio test (SPRT), the sample size is

(i) fixed

(ii) fixed but small

(iii) fixed but large

(iv) a random variable

2. Answer the following questions briefly : 2×7=14

(a) Mention the criterion that should be satisfied by a good estimator.

(b) State the sufficient condition for consistency.

(c) Compare the method of moments with maximum likelihood estimator.

(d) Define most powerful critical region.

(e) State the properties of likelihood ratio test.

(f) State two applications of sequential test.

(g) Define operating characteristic function of a test.

3. (a) (i) Let $X_1, X_2, \ldots, X_n$ be a random sample from binomial population with parameters n and p. If p is known, find unbiased estimator of n and variance of the estimator. 1+4=5

(ii) State the Lehmann-Scheffe theorem. Let $(X_1, X_2, \ldots, X_n)$ be a random sample from $N(\mu, \delta^2)$ population. Find sufficient statistics for μ and δ^2. 1+5=6

Or

(b) (i) What do you mean by minimum variance unbiased estimator?

A random sample $X_1, X_2, \ldots, X_n$ is taken from a normal population with mean zero and variance δ^2. Examine if $\frac{\Sigma X_i^2}{n}$ is an MVB estimator for δ^2.

1+4=5

(ii) Give the statement of Cramer-Rao inequality. Suppose X follows a continuous uniform distribution

$$f(x, \theta) = \begin{cases} \frac{1}{2}\theta; & -\theta < x < \theta, \ -\alpha < \theta < \alpha \\ 0; & \text{otherwise} \end{cases}$$

show that family of distribution is not complete. 2+4=6

4. (a) State the invariance property of maximum likelihood estimator (MLE). Find the maximum likelihood estimator for the parameter θ of a Poisson distribution on the basis of a sample of size n. 1+4=5

Or

(b) Let X be a binomial random variable with parameters n and p, where $f_1(p) = 1$, $0 < p < 1$. Find Bayes estimator of p. 5

5. Answer any *two* of the following : 5×2=10

(a) What is statistical hypothesis? Define null and alternative hypotheses with examples. Define level of significance. 1+3+1=5

(b) State the utility of Neyman-Pearson lemma. Show that power of a BCR (Best Critical Region) never less than its size. 1+4=5

(c) Under what circumstance would you recommend likelihood ratio test? Discuss the general method of construction of likelihood ratio test. 1+4=5

6. (a) Describe Wald's sequential probability ratio test. How large must n be for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$ for a normal population with mean θ and variance 1? 5+4=9

Or

(b) Use SPRT to test the hypothesis $H_0 : \theta = \theta_0$ vs. $H_1 : \theta = \theta_1$ for the Poisson distribution having p.m.f.

$$f(x, \theta) = \frac{e^{-\theta}\theta^x}{x!}; \ x = 0, 1, 2, \ldots$$

Also find OC and ASN functions. 9

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

1. Select the correct one out of the given alternatives : 1×8=8

(a) Bias of an estimator can be

(i) positive

(ii) negative

(iii) either positive or negative

(iv) always zero

(b) Factorization theorem for sufficiency is known as

(i) Rao-Blackwell theorem

(ii) Cramer-Rao theorem

(iii) Chapman-Robbins theorem

(iv) Fisher-Neyman theorem

(c) A maximum likelihood estimator is always

(i) unbiased

(ii) consistent

(iii) unbiased and consistent

(iv) consistent but need not be unbiased

(d) The error or rejecting H_0 when H_0 is true is called

(i) type–II error

(ii) type–I error

(iii) sampling error

(iv) None of the above

(e) The size of critical region is

(i) α

(ii) β

(iii) $1-\beta$

(iv) $1-\alpha$

(f) Neyman-Pearson lemma provides

(i) a most powerful test

(ii) an unbiased test

(iii) an admissible test

(iv) a minimax test

of simple hypothesis against a simple alternative hypothesis.

(g) The decision criteria in SPRT depend on

(i) type–I error

(ii) type–II error

(iii) Both (i) and (ii)

(iv) None of the above

(h) In SPRT, the sample size is

(i) fixed but large

(ii) fixed but small

(iii) a random variable

(iv) None of the above

2. Answer the following in brief : 2×5=10

(a) Define parameter space with an example.

(b) Mention the criteria that should be satisfied by a good estimator.

(c) Define level of significance and power of a test.

(d) Define most powerful test.

(e) State two applications of sequential test.

3. (a) (i) Let $X_1, X_2, ..., X_n$ be a random sample of size n from a Poisson population with parameter λ. Find the unbiased estimators of λ and λ^2. 5

(ii) Prove that in sampling from a $N(\mu, \sigma^2)$ population, the sample mean is a consistent estimator for μ. 4

Or

(b) (i) If $(X_1, X_2, ..., X_n)$ be a random sample from a population with p.d.f.

$$f(x; \theta) = \theta x^{\theta-1}; \ 0 < x < 1, \ \theta > 0$$

show that $t_1 = \prod_{i=1}^{n} X_i$ is sufficient for θ. 4

(ii) Obtain the minimum variance bound estimator (MVBE) for μ of the normal population $N(\mu, \sigma^2)$ where σ^2 is known. 5

4. (a) (i) Find the maximum likelihood estimator for p in the Bernoulli's probability distribution

$$f(x) = p^x(1-p)^{1-x};\ x = 0, 1$$ 4

Or

(ii) Prove that if a sufficient estimator exists, it is a function of MLE.

(b) Write a short note on any *one* of the following : 3

(i) Method of moments

(ii) Method of minimum chi-square

5. (a) Explain the concepts of type–I and type–II errors, critical region and best critical region. State Neyman-Pearson lemma for testing a simple null hypothesis against a simple alternative hypothesis. 8

Or

(b) Write a note on likelihood ratio test. State the properties of likelihood ratio test.

6. (a) Give the SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$ in sampling from $N(\theta, \sigma^2)$, σ^2 being known. Also obtain OC and ASN functions. 8

Or

(b) Describe SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ for

$$f(x, \theta) = \theta^x(1-\theta)^{1-x};\ x = 0, 1;\ 0 < \theta < 1$$

Write down the approximate expressions for OC and ASN functions of the test.

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