

Total No. of Printed Pages—11

4 SEM TDC STSH (CBCS) C 9 (N/O)

2 0 2 4

(May/June)

STATISTICS

(Core)

Paper : C-9

(**Linear Models**)

*The figures in the margin indicate full marks
for the questions*

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the given alternatives in each question : 1×5=5

(a) In the linear model, $y = X\beta + \epsilon$

(i) $E(\epsilon) = 0$ for all X and $\text{cov}(\epsilon) = \sigma^2 I$

(ii) $E(\epsilon) = 1$ and $\text{cov}(\epsilon) = 0$

(iii) $E(\epsilon) > 1$ and $\text{cov}(\epsilon) = 1$

(iv) None of the above

(2)

- (b) In simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, $i = 1, 2, \dots, n$, the value of $\hat{\beta}_0$ is

(i) $\hat{\beta}_0 = \bar{y} + \bar{x}$

(ii) $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

(iii) $\hat{\beta}_0 = x_i y_i + \hat{\beta}_1 \bar{x}$

(iv) None of the above

- (c) The term analysis of variance was introduced by

(i) R. A. Fisher

(ii) Kolmogorov

(iii) Gauss-Markov

(iv) None of them

- (d) ANOVA technique is designed to

(i) compare the homogeneity of several means simultaneously

(ii) test the equality of several population variances

(iii) Both (i) and (ii)

(iv) Neither (i) nor (ii)

(3)

- (e) The assumption of constant variance of the residual is known as

(i) autocorrelation

(ii) heteroscedasticity

(iii) homoscedasticity

(iv) multicollinearity

2. Answer the following questions in brief :

2×5=10

- (a) What do you mean by estimable function?

- (b) State the assumptions associated with multiple regression model.

- (c) What do you mean by concomitant variable?

- (d) Write some applications of analysis of variance.

- (e) Mention any two methods which are commonly used as model adequacy checking.

3. (a) Write the properties of estimability of linear parametric function. Show that least squares estimators are unbiased.

$$3\frac{1}{2} + 3\frac{1}{2} = 7$$

Or

- (b) Estimate the error variance or residual sum of squares of the general linear model in matrix form.

7

4. (a) What are the assumptions of the simple linear regression model? For a simple linear regression model, $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, find the $100(1-\alpha)\%$ confidence interval for the mean response at a particular value of the regressor variable X .

$$2+5=7$$

Or

- (b) For multiple linear regression model, with usual notation in matrix form, if $E(y) = X\beta$ and $\text{cov}(y) = \sigma^2 T$, show that the least square estimators $\hat{\beta}$ have minimum variance among all linear unbiased estimators.

7

5. For a simple linear regression model, show that $\hat{\beta}_0$ is an unbiased estimator of β_0 .

5

6. Answer any two of the following :

- (a) What do you mean by analysis of variance? State the assumptions of analysis of variance. Distinguish between fixed effect and random effect models.

$$1+2+4=7$$

- (b) Describe analysis of variance of one-way classification of fixed effect model, stating clearly—

(i) the hypothesis to be tested;

(ii) the assumptions to be made;

(iii) the least squares estimates of the parameter;

(iv) the partitioning of sum of squares with d.f.;

(v) the test statistics to be used;

(vi) the ANOVA table. $1+1+1+1+1+2=7$

- (c) What is two-way classification with one observation per cell in ANOVA? Give the mathematical model and the underlying assumptions.

$$2+2+3=7$$

(6)

- (d) Derive the analysis of covariance for a one-way layout (with one concomitant variable only).

7

7. (a) What is multicollinearity? Explain the different sources of multicollinearity.

2+5=7

Or

- (b) Write an explanatory note on quantile-quantile plot.

7

(7)

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

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- (b) In simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, $i = 1, 2, \dots, n$, the value of $\hat{\beta}_0$ is

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(b) Estimate the error variance or residual sum of squares of the general linear model in matrix form.

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- (iii) the least squares estimates of the parameter;
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(v) the test statistics to be used;
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- (c) What is two-way classification with one observation per cell in ANOVA? Give the mathematical model and the underlying assumptions. 2+2+3=7

- (d) Derive the analysis of covariance for a one-way layout (with one concomitant variable only). 7

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Total No. of Printed Pages—9

4 SEM TDC STSH (CBCS) C 10 (N/O)

2024

(May/June)

STATISTICS

(Core)

Paper : C-10

(Statistical Quality Control)

*The figures in the margin indicate full marks
for the questions*

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following
alternatives : 1×5=5

(a) Quality

(i) is inversely proportional to
variability

(ii) means fitness for use

(iii) is the reduction of variability in
process and product

(iv) All of the above

(2)

- (b) Variation due to assignable causes in the product occurs due to
- (i) faulty process
 - (ii) carelessness of operators
 - (iii) poor quality of raw material
 - (iv) All of the above
- (c) Which probability distribution is used for $\bar{x}$ and R chart?
- (i) Normal
 - (ii) Binomial
 - (iii) Poisson
 - (iv) None of the above
- (d) The graph of the proportion of defectives in the lot against average sample number is
- (i) OC curve
 - (ii) ASN curve
 - (iii) power curve
 - (iv) All of the above
- (e) The starting point for statistical six-sigma analysis is
- (i) Juran's 80/20 rule
 - (ii) standard deviation
 - (iii) defects per million opportunities
 - (iv) None of the above

24P/1296

(Continued)

(3)

2. Answer the following questions in brief :

3×4=12

- (a) Write a note on seven tools of SPC, used in statistical process-control.
- (b) Which control charts are used for sampling by attributes? Give practical examples where these charts can be used.
- (c) Define producer's risk and consumer's risk in statistical quality control.
- (d) Define CTQ (Critical to Quality) characteristic.

3. Answer any one of the following :

10

- (a) Explain the concept of quality. Describe the eight dimensions of quality in brief.

2+8=10

- (b) Discuss the rational sub-group concept. What part does it play in control chart analysis? What are the warning limits on a control chart? How can they be used?

3+3+2+2=10

- (c) Explain the justification for using the 3-8 limits in the control charts irrespective of the actual probability distribution of the quality characteristic.

5+5=10

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(Turn Over)

(4)

4. (a) Explain in detail the $\bar{X}$ and R -charts. State the control limits of S -chart and when is S -chart used in place of R -chart. $6+3+2=11$

Or

- (b) Distinguish between defects and defective. Give some examples of defects for which C -chart is applicable. How do you calculate control limits for C -chart? Discuss the assumptions and approximations involved in the calculations. $2+2+5+2=11$

5. (a) Explain what is single sampling plan and double sampling plan. Discuss the merits and demerits of single and double sampling plan. 6

Or

- (b) What are average sample number (ASN) and average total inspection (ATI)? Explain the method of their calculation for single sampling plan. 6

6. (a) What do you mean by acceptance sampling procedure? Explain the following terms in single sampling plan : 5

(i) AOQL

(ii) LTPD

Or

- (b) Delineate operating characteristic (OC) curve of a sampling plan. 5

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(Continued)

(5)

7. (a) What do you mean by VOC? Write about the necessities of voice of customers as a basic concept of six-sigma methodology. Mention some different ways of capturing VOC. $2+2+2=6$

Or

- (b) What is lean six-sigma? What are the benefits of lean six-sigma in the industrial world? What are 5 S's in lean approach to organize a workspace? $2+2+2=6$

24P/1296

(Turn Over)

(6)

(Old Course)

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24P/1296

(Continued)

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24P/1296

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