

Total No. of Printed Pages—12

6 SEM TDC STS H (CBCS) C 14 (N/O)

2025

(May)

STATISTICS

(Core)

Paper : C-14

(Multivariate Analysis and Non-Parametric Methods)

*The figures in the margin indicate full marks
for the questions*

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following alternatives :

1×6=6

- (a) If the joint distribution of the variables X and Y is BVN $(0, 0, 1, 1, \rho)$, then the correlation coefficient between X^2 and Y^2 is equal to

(i) 1

(ii) -1

(iii) ρ^2

(iv) 0

(2)

- (b) A measure of linear association of a variable say X_1 with a number of other variables $X_2, X_3, \dots, X_k$ is known as
- (i) partial correlation
 - (ii) multiple correlation
 - (iii) simple correlation
 - (iv) autocorrelation
- (c) Principal component analysis is meant for
- (i) reducing the number of observed variables
 - (ii) increasing the number of variables
 - (iii) reducing the correlation between variables
 - (iv) reducing the variance among variables
- (d) Factor analysis is a technique in which the observed variables are expressed in terms of a
- (i) factor
 - (ii) linear combination of factors
 - (iii) correlation
 - (iv) distance

(3)

- (e) A sequence of symbols shows lack of randomness if there are
- (i) too many runs
 - (ii) too few runs
 - (iii) Both (i) and (ii)
 - (iv) Neither (i) nor (ii)
- (f) Kolmogorov test is based on the theorem
- (i) N. V. Smirnov
 - (ii) A. N. Kolmogorov
 - (iii) Kolmogorov-Smirnov
 - (iv) Glivenko-Cantelli

2. Answer the following questions in brief : $2 \times 6 = 12$

- (a) State the properties of random vector.
- (b) Define the probability density function of multivariate normal distribution.
- (c) State the features of principal component analysis.
- (d) State the assumptions underlying discriminant analysis.
- (e) What are the assumptions of non-parametric test?
- (f) How do you solve the problem of zero differences in sign test?

3. (a) State the properties of bivariate normal distribution.

Let the joint p.d.f. of X and Y be

$$f(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[\left(\frac{x-\mu_1}{\sigma_1} \right)^2 - 2\rho \left(\frac{x-\mu_1}{\sigma_1} \right) \left(\frac{y-\mu_2}{\sigma_2} \right) + \left(\frac{y-\mu_2}{\sigma_2} \right)^2 \right] \right\}$$

where $-\infty < x < \infty$, $-\infty < y < \infty$, $-1 < \rho < 1$.

Find—

- (i) the conditional distribution of $X | Y = y$;

- (ii) $P(5 < X < 9 | Y = 6)$. Given that $\mu_1 = 3$, $\mu_2 = 4$, $\sigma_1^2 = 16$, $\sigma_2^2 = 25$, $\rho = 0.8$, $\phi(1.97) = 0.47$ and $\phi(0.3) = 0.11791$. $2+3+2=7$

Or

- (b) How do you express and organize multivariate data?

Let

$$X = \begin{bmatrix} 10 & 100 \\ 12 & 110 \\ 11 & 105 \end{bmatrix}$$

Calculate the mean vector and variance-covariance matrix. $3+4=7$

4. Answer any two from the following questions :

$6 \times 2 = 12$

- (a) If $X \sim N_p(\mu, \Sigma)$, then show that the components of X will be jointly independent if and only if the covariance of X_j and X_k ($j \neq k = 1, 2, \dots, p$) is zero. 6
- (b) State and prove the reproductive property of multivariate normal distribution. $1+5=6$
- (c) Explain the concept of partial correlation coefficient with examples. Show that partial correlation coefficient is the geometric mean between the regression coefficient. $3+3=6$
- (d) Define multiple correlation with example. Discuss the different properties of multiple correlation. $3+3=6$

5. (a) For what purposes discriminant analysis is used? How to obtain discriminant scores? Explicate Fisher's linear discriminant function. $2+2+5=9$

Or

- (b) What are the uses of principal component analysis? Distinguish between principal component and factor analysis. Deliberate the basic approach of factor analysis. $2+2+5=9$

(6)

6. (a) What do you mean by empirical distribution function? Write some applications of Kolmogorov-Smirnov test. Explicate Mann-Whitney U-test for testing the identicalness of two populations.

2+2+5=9

Or

- (b) State the assumptions of Kruskal Wallis test. Discuss the Kruskal Wallis method of analysis for one-way classification of data.

3+6=9

(7)

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

1. Choose the correct answer from the following alternatives : 1×5=5

- (a) If the joint distribution of the variables X and Y is BVN $(0, 0, 1, 1, \rho)$, then the correlation coefficient between X^2 and Y^2 is equal to

(i) 1

(ii) -1

(iii) ρ^2

(iv) 0

- (b) A measure of linear association of a variable say X_1 with a number of other variables $X_2, X_3, \dots, X_k$ is known as

(i) partial correlation

(ii) multiple correlation

(iii) simple correlation

(iv) autocorrelation

(c) Principal component analysis is meant for

- (i) reducing the number of observed variables
- (ii) increasing the number of variables
- (iii) reducing the correlation between variables
- (iv) reducing the variance among variables

(d) A sequence of symbols shows lack of randomness if there are

- (i) too many runs
- (ii) too few runs
- (iii) Both (i) and (ii)
- (iv) Neither (i) nor (ii)

(e) Kolmogorov test is based on the theorem

- (i) N. V. Smirnov
- (ii) A. N. Kolmogorov
- (iii) Kolmogorov-Smirnov
- (iv) Glivenko-Cantelli

2. Answer the following questions in brief : $2 \times 5 = 10$

- (a) State the properties of random vector.
- (b) Define the probability density function of multivariate normal distribution.
- (c) State the features of principal component analysis.
- (d) What are the assumptions of non-parametric test?
- (e) How do you solve the problem of zero differences in sign test?

3. (a) State the properties of bivariate normal distribution.

Let the joint p.d.f. of X and Y be

$$f(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[\left(\frac{x-\mu_1}{\sigma_1} \right)^2 - 2\rho \left(\frac{x-\mu_1}{\sigma_1} \right) \left(\frac{y-\mu_2}{\sigma_2} \right) + \left(\frac{y-\mu_2}{\sigma_2} \right)^2 \right] \right\}$$

where $-\infty < x < \infty$, $-\infty < y < \infty$, $-1 < \rho < 1$.

Find—

- (i) the conditional distribution of $X | Y = y$;

- (ii) $P(5 < X < 9 | Y = 6)$. Given that
 $\mu_1 = 3$; $\mu_2 = 4$, $\sigma_1^2 = 16$, $\sigma_2^2 = 25$,
 $\rho = 0.8$, $\phi(1.97) = 0.47$ and
 $\phi(0.3) = 0.11791$. 2+3+2=7

Or

- (b) How do you express and organize multivariate data?
 Let

$$X = \begin{bmatrix} 10 & 100 \\ 12 & 110 \\ 11 & 105 \end{bmatrix}$$

Calculate the mean vector and variance-covariance matrix. 3+4=7

4. Answer any two from the following questions : 6×2=12

- (a) If $X \sim N_p(\mu, \Sigma)$, then show that the components of X will be jointly independent if and only if the covariance of X_j and X_k ($j \neq k = 1, 2, \dots, p$) is zero. 6

- (b) State and prove the reproductive property of multivariate normal distribution. 1+5=6

- (c) Explain the concept of partial correlation coefficient with examples. Show that partial correlation coefficient is the geometric mean between the regression coefficient. 3+3=6

- (d) Define multiple correlation with example. Discuss the different properties of multiple correlation. 3+3=6

5. (a) For what purposes discriminant analysis is used? How to obtain discriminant scores? Explicate Fisher's linear discriminant function. 2+2+4=8

Or

- (b) What are the uses of principal component analysis? Distinguish between principal component and factor analysis. Deliberate the basic approach of factor analysis. 2+2+4=8

6. (a) What do you mean by empirical distribution function? Write some applications of Kolmogorov-Smirnov test. Explicate Mann-Whitney U-test for testing the identicalness of two populations. 2+2+4=8

Or

- (b) State the assumptions of Kruskal-Wallis test. Discuss the Kruskal-Wallis method of analysis for one-way classification of data. $3+5=8$

★ ★ ★

Total No. of Printed Pages—9

6 SEM TDC STSH (CBCS) C 13 (N/O)

2025

(May)

STATISTICS

(Core)

Paper : C-13

(Design of Experiments)

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Choose the correct answer from the following : 1×5=5

(a) When all experimental units are homogeneous, the most suitable design for an experiment is

- (i) CRD
- (ii) RBD
- (iii) LSD
- (iv) None of the above

(2)

- (b) Randomization is a process in which the treatments are allocated to the experimental units
- (i) at the will of the investigator
 - (ii) in a sequence
 - (iii) with equal probability
 - (iv) None of the above
- (c) Latin square design controls
- (i) two-way variation
 - (ii) three-way variation
 - (iii) multi-way variation
 - (iv) no variation
- (d) A BIBD is said to be symmetrical, if number of blocks =
- (i) number of factors
 - (ii) number of treatments
 - (iii) number of levels
 - (iv) number of degrees of freedom

P25/964

(Continued)

(3)

- (e) In 2^2 -factorial design, the total treatment combinations are
- (i) 2
 - (ii) 4
 - (iii) 6
 - (iv) None of the above

2. Answer the following in brief : $2 \times 7 = 14$

- (a) What are the requirements of a good experimental design?
 - (b) What purpose does replication serve in experimental design?
 - (c) What are the drawbacks of a CRD?
 - (d) State the situation when the missing plot technique arises.
 - (e) Define orthogonal contrast.
 - (f) Explain symmetrical and asymmetrical factorial experiments.
 - (g) Distinguish between total confounding and partial confounding.
3. (a) Give the statistical model and the hypothesis to be tested in an RBD. Also obtain the relative efficiency of an RBD over CRD. $4+6=10$

P25/964

(Turn Over)

(4)

Or

- (b) What do you mean by a Latin square design? Give the assumptions and applications of an LSD. Discuss how randomization is done in such a design. Find the efficiency of this design in comparison to an RBD.

$$1+3+2+4=10$$

4. (a) Define Balanced Incomplete Block Design (BIBD) and derive the equality and inequality relations among its parameters.

$$3+2+2+2=9$$

Or

- (b) (i) When is a Balanced Incomplete Block Design (BIBD) called symmetric?

3

- (ii) Show that in a symmetric BIBD, any two blocks have the same number λ of treatments in common.

4

- (iii) Indicate when a BIBD becomes resolvable and affine resolvable.

2

5. (a) What is factorial experiment? A complete 2^3 -experiment is replicated in r times. Describe the procedure for testing the presence of different main effects and interactions.

$$3+5=8$$

P25/964

(Continued)

(5)

Or

- (b) What is treatment contrast? When are two such contrasts said to be orthogonal? Show that in an RBD, every block contrast is orthogonal to every treatment contrast. Show that in a 2^3 -experiment, the main effects and interaction effects are mutually orthogonal.

$$2+1+2+3=8$$

6. (a) In a completely confounded design, the information on the confounded effects are totally lost. Explain how these informations are recovered to some extent with the help of a partially confounded factorial design.

9

Or

- (b) Explain the principle of confounding in design of experiments. Suppose we have a 2^3 -experiment with three factors each at two levels. Suppose there are three replicates each divided into 2 blocks, each block of 4 units. Show how you will confound ABC, in the first replication, AC in the second replication and BC in the third replication.

P25/964

(Turn Over)

(6)

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. Select the correct alternative from the given options : 1×5=5

(a) In ANOVA, the sum of squares divided by its degrees of freedom, gives the corresponding

(i) variance

(ii) standard deviation

(iii) standard error

(iv) None of the above

(b) Which of the following is true for a Balanced Incomplete Block Design (BIBD)?

(i) $br = vk, \lambda(v-1) = r(k-1); b \geq v$

(ii) $bk = vr, \lambda(v-1) = r(k-1); b \geq v$

(iii) $bk = vr, \lambda(r-1) = v(k-1); b \geq v$

(iv) $br = vk, \lambda(k-1) = r(v-1); b \geq v$

P25/964

(Continued)

(7)

(c) In 2^2 -factorial design, the total treatment combinations are

(i) 2

(ii) 4

(iii) 6

(iv) None of the above

(d) Replication in an experiment means

(i) the number of blocks

(ii) the total number of treatments

(iii) the number of times a treatment occurs in an experiment

(iv) None of the above

(e) If the interactions AB and BC are confounded with incomplete block in a 2^n -factorial experiment, then automatically confounded effect is

(i) ABC

(ii) AC

(iii) A

(iv) C

P25/964

(Turn Over)

2. Answer any *five* from the following questions in brief : $3 \times 5 = 15$

(a) Explain the meaning of 'analysis of variance' and give its uses. State the basic assumptions in the analysis of variance.

(b) How is the principle of randomization used in CRD?

(c) Define BIBD. When is a BIBD said to be symmetrical?

(d) Prove that for a BIBD with parameters v, r, b, k and λ :

$$b \geq v + r - k$$

(e) Explain symmetrical and asymmetrical factorial experiments.

(f) Differentiate between complete confounding and partial confounding.

(g) When do you call the partial confounding as balanced and unbalanced confounding?

3. Answer any *three* from the following questions : $10 \times 3 = 30$

(a) Give the layout and analysis of a completely randomized block design and explain the situations when it is used. Discuss its demerits. $6 + 2 + 2 = 10$

(b) Explain how the principles of replication, randomization and local control are used in randomized block design. Obtain the formula for estimating two missing values in an RBD with p treatments and q blocks. $5 + 5 = 10$

(c) Explain the missing plot technique and state the situation when it arises. Apply the missing plot technique to the case when a plot of yield is missing in an RBD. $5 + 5 = 10$

(d) Define main effect and interaction in factorial experiments. A complete 2^3 -experiment is replicated r times. Describe the procedure for testing the presence of different main effects and interactions. $5 + 5 = 10$

(e) What is treatment contrast? When are two such contrasts said to be orthogonal? Show that in a 2^3 -experiment, the main effects and interaction effects are mutually orthogonal. How would you obtain the SS (sums of square) due to main effect or interaction effect in a 2^3 -experiment? $3 + 3 + 4 = 10$
