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5 SEM TDC STSH (CBCS) C 11 (N/O)

2025

(Nov/Dec)

STATISTICS

(Core)

Paper : C–11

(Stochastic Processes and Queuing Theory)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the following alternatives : $1 \times 6 = 6$

(a) What is the term for the probability of transition from one state to another in a Markov chain?

(i) State probability

(ii) Transition probability

(iii) Joint probability

(iv) Conditional probability

(b) A random walk is an example of

(i) a continuous-time, continuous-state stochastic process

(ii) a discrete-time, discrete-state stochastic process

(iii) a continuous-time, discrete-state stochastic process

(iv) a discrete-time, continuous-state stochastic process

(c) Higher transition probabilities can be computed by

(i) spectral decomposition method

(ii) Chapman-Kolmogorov equation

(iii) generating function method

(iv) All of the above

(d) Which of the following statements is not correct?

(i) An absorbing state is recurrent

(ii) An ergodic state is recurrent

(iii) Recurrent state is always periodic

(iv) An absorbing state is aperiodic

(e) A persistent state of a Markov chain is said to be null persistent if its mean recurrence time is

(i) finite

(ii) infinite

(iii) zero

(iv) one

(f) In queuing theory, what does the '1' represent in the notation $M/M/1$?

(i) One customer in the system

(ii) One server

(iii) One queue

(iv) One minute service time

2. Answer the following questions in brief : 2×6=12

(a) Derive the p.g.f. of geometric distribution and hence find its mean and variance.

(b) Give some examples of continuous time discrete state space stochastic process.

(c) A particle performs a random walk with absorbing barriers, say as 0 and 4. Whenever it is at any positive $r\ (0 < r < 4)$; it moves to $(r+1)$ with probability p or to $(r-1)$ with probability q, $p+q=1$. But as soon as it reaches 0 or 4 it remains there itself. Find the transition probability matrix.

(d) Distinguish between irreducible and reducible Markov chain.

(e) State the characteristics of Yule-Furry process.

(f) What do the letters in the symbolic representation $(a/b/c):d/e$ of a queuing model represent?

3. (a) (i) Define bivariate probability generating function. Given the bivariate p.g.f of (X, Y) as

$$P(S_1, S_2) = \exp[-a-b-c+aS_1+bS_2+cS_1S_2]$$

Find the p.g.f of $X+Y$ and identify the distribution. 1+1=2

(ii) The r.v. X has logarithmic series distribution if

$$p_k = P(X=k) = \frac{Xq^k}{k};\ k = 1, 2, 3, \cdots$$

$$X - \frac{1}{\log p};\ 0 < q = 1-p < 1$$

Find the p.g.f of X. Also, find mean and variance. 3

Or

(b) Define stochastic process and give two examples of continuous time continuous state space stochastic process. Consider the process

$$\{X(t),\ t > 0\}$$

$$X(t) = A_0 + A_1 t + A_2 t^2$$

where A_0, A_1 and A_2 are uncorrelated random variables with mean 0 and variance σ^2. Check whether the given process is covariance stationary or not. 1+1+3=5

4. Answer any *four* questions from the following : 4×4=16

(a) Prove that transition probability completely specifies the distribution of Markov chain.

(b) Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is $\frac{1}{3}$ and the probability of a rainy day following a dry day is $\frac{1}{2}$.

(i) If May 1 is a dry day, what is the probability that May 3 is a dry day?

(ii) What is the probability that May 5 is a dry day? 2+2=4

(c) Define Markov chain. Let $\{X_n, n \geq 0\}$ be a Markov chain with t.p.m.

$$P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}; \text{ state space} = \{0, 1, 2\}$$

and initial distribution

$$P\{X_0 = i\} = \frac{1}{3}; \ i = 0, 1, 2$$

Find—

(i) $P\{X_2 = 2, X_1 = 1, X_0 = 2\}$;

(ii) $P\{X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2\}$;

(iii) $P\{X_2 = 2, X_1 = 1 | X_0 = 2\}$.

(d) State and prove Chapman-Kolmogorov theorem.

(e) Let $\{X_n : n \geq 0\}$ be a Markov chain with t.p.m.

$$P = \begin{bmatrix} 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2}{4} & 0 & \frac{2}{4} \\ \frac{2}{4} & \frac{2}{4} & 0 & 0 \end{bmatrix}; \text{ state space} = \{1, 2, 3, 4\}$$

Examine the nature of states of the chain, i.e., transient or recurrent.

5. (a) What is birth-death process? Set up the differential-difference equation(s) for $P_n(t)$ in the case of pure birth process. 2+5=7

Or

(b) Show that if the intervals between successive occurrences of an event E are independently distributed with mean $\frac{1}{\lambda}$, then the events E form a Poisson process with mean λt. 7

6. (a) State Little's formula. Draw the state transition diagram of $M/M/1 : N/FIFO$ model, N being infinite. Derive the

steady-state solution $M/M/1:\infty/\text{FIFO}$ model. Also find the expected length system and expected length of queue.

2+2+3+2=9

Or

(b) Define a queuing system. Discuss the basic features which characterize a queuing system.

"Queue is a management of congestions." Elucidate the statement.

5+4=9

(Old Course)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

1. Choose the correct answer from the following alternatives : 1×5=5

(a) What is the term for the probability of transition from one State to another in a Markov chain?

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(iii) Joint probability

(iv) Conditional probability

(b) A random walk is an example of

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$$p_k = P(X=k) = \frac{Xq^k}{k};\ k=1, 2, 3, \cdots$$

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where A_0, A_1 and A_2 are uncorrelated random variables with mean 0 and variance σ^2. Check whether the given process is covariance stationary or not. 1+1+3=5

4. Answer any *three* questions from the following : 4×3=12

(a) Prove that transition probability completely specifies the distribution of Markov chain.

(b) Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is $\frac{1}{3}$ and the probability of a rainy day following or dry day is $\frac{1}{2}$.

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(ii) What is the probability that May 5 is a dry day? 2+2=4

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(d) State and prove Chapman-Kolmogorov theorem.

(e) Let $\{X_n : n \geq 0\}$ be a Markov chain with t.p.m.

$$P = \begin{bmatrix} 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2}{4} & 0 & \frac{2}{4} \\ \frac{2}{4} & \frac{2}{4} & 0 & 0 \end{bmatrix}; \text{ state space} = \{1, 2, 3, 4\}$$

Examine the nature of states of the chain, i.e., transient or recurrent.

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