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2 SEM TDC MTMH (CBCS) C 3

2026

(May/June)

MATHEMATICS

(Core)

Paper : C–3

(Real Analysis)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions.

1. *(a)* Using algebraic properties of $\mathbb{R}$, prove that if u and $b \neq 0$ are elements in $\mathbb{R}$ with $u \cdot b = b$, then $u = 1$. 2

(b) State True or False : 1

There exists a rational number whose square is 2.

(c) Fill in the blank : 1

The order properties of $\mathbb{R}$ refer to the notion of positivity and _____ between real numbers.

(d) If $ab > 0$, then prove that either (i) $a > 0$ and $b > 0$ or (ii) $a < 0$ and $b < 0$. 2+2=4

Or

Prove that if x is a rational number and y is an irrational number, then $x + y$ is an irrational number. If, in addition, $x \neq 0$, then show that xy is an irrational number. 4

(e) If $S = \left\{-2, -\frac{3}{2}, -\frac{4}{3}, \ldots, -\frac{n+1}{n}\right\}$, then find $\inf S$ and $\sup S$ if they exist. 2

(f) Prove that the interval $[0, 1]$ is uncountable. 4

Or

Prove that Cartesian product of two countable sets is countable.

(g) If A and B are two non-empty subsets of $\mathbb{R}$ satisfying the property $a \leq b$ for all $a \in A$ and all $b \in B$, then prove that $\sup A \leq \inf B$. 4

Or

If $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, then show that $\inf S = 0$.

(h) Let $I_n = \left[0, \frac{1}{n}\right]$ for $n \in \mathbb{N}$. Prove that

$$\bigcap_{n=1}^{\infty} I_n = \{0\}$$ 3

(i) State and prove the nested interval property. 5

Or

Prove that the set of natural numbers is not bounded above.

(j) Let S be a non-empty subset of $\mathbb{R}$ and let $a \in \mathbb{R}$ be arbitrary. Define $a + S = \{a + s : s \in S\}$. Prove that $\sup\{a + S\} = a + \sup S$. 4

2. (a) State True or False : 1

The number of terms in a sequence is always infinite.

(b) Consider the sequence $< S_n >$, where $S_n = n^2$, $n \in \mathbb{N}$.

(i) Is the sequence bounded above or bounded below?

(ii) Write the limit point of the sequence if it has any. 2+1=3

(c) Every bounded sequence has a limit point. Is the converse true? Justify. 1+2=3

(d) State the monotone convergence theorem. 1

(e) Show that $\underset{n\to\infty}{\text{Lt}}\ b^n = 0$, if $0 < b < 1$. 4

Or

Prove that $\underset{n\to\infty}{\text{Lt}}\ \frac{1}{1+na} = 0$, $a > 0$.

(f) Let $X = < x_n >$ be defined as $x_1 = 1$, $x_{n+1} = \frac{1}{4}x_n + 2$, $n \geq 1$. Show that $< x_n >$ is monotone and bounded. 4

(g) Prove that $\underset{n\to\infty}{\text{Lt}}\ x_n = 0$ if and only if $\underset{n\to\infty}{\text{Lt}}\ (|x_n|) = 0$. 4

(h) Discuss the convergence or divergence of the following sequences (any *one*) : 4

(i) Sequence $< x_n >$, where

$$x_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

(ii) Sequence $< x_n >$, where

$$x_n = \frac{2n^2+3}{n^2+1}$$

(i) Define Cauchy sequence. 1

(j) Show that every Cauchy sequence is bounded. Is the converse true? 4+1=5

Or

Establish the convergence or divergence of the sequence

$$y_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n}, \quad n \in \mathbb{N}$$ 5

3. (a) Discuss the convergence of a geometric series. 4

(b) State the condition under which the *p*-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges. 1

(c) Suppose that $X = < x_n >$ and $Y = < y_n >$ are strictly positive sequences and suppose that the following limit exists in $\mathbb{R}$:

$$r = \underset{n\to\infty}{\text{Lt}} \left(\frac{x_n}{y_n} \right)$$

Prove that if $r \neq 0$, then $\sum_{n=1}^{\infty} x_n$ is convergent if and only if $\sum_{n=1}^{\infty} y_n$ is convergent. 4

Or

Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$$

(d) Define : 1×3=3

(i) Positive term series

(ii) Alternating series

(iii) Conditionally convergent series

(e) Test the convergence of the following (any *two*) : 4×2=8

(i) $1 + \frac{4}{2!} + \frac{4^2}{3!} + \frac{4^3}{4!} + \cdots$

(ii) $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \cdots$

(iii) $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

(iv) $\frac{1}{x} - \frac{1}{x+a} + \frac{1}{x+2a} - \frac{1}{x+3a} + \cdots$

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