Total No. of Printed Pages—12

2 SEM TDC STSH (CBCS) C 4 (N/O)

2026

(May/June)

STATISTICS

(Core)

Paper : C–4

(Algebra)

The figures in the margin indicate full marks for the questions

(New Course)

Full Marks : 55

Pass Marks : 22

Time : 3 hours

1. Choose the correct answer from the given alternatives in each question : 1×6=6

(a) A polynomial of degree n has how many complex roots?

(i) Exactly n

(ii) $n-1$

(iii) At least $n+1$

(iv) None of the above

(b) A set containing only the zero vector is always

(i) linearly dependent

(ii) linearly independent

(iii) member of any bias

(iv) None of the above

(c) The trace of a square matrix A, denoted by tr (A), is defined as the

(i) product of diagonal elements

(ii) sum of diagonal elements

(iii) sum of all elements in the matrix

(iv) determinant of the matrix

(d) What happens to the value of a determinant if all its rows are interchanged with its columns?

(i) The value remains unchanged

(ii) The sign of the value changes

(iii) The value is multiplied by -1

(iv) The value becomes zero

(e) The determinant of a skew-symmetric matrix of odd order is always

(i) 0

(ii) 1

(iii) -1

(iv) a perfect square

(f) The rank of a null matrix is always

(i) 0

(ii) 1

(iii) infinity

(iv) undefined

2. Answer the following questions : 3×5=15

(a) Find the values of A, B and C for which

$$A(x-3)(x-1)+B(x+1)(x-1)+C(x+1)(x-3)=6x-10$$

is an identity.

(b) Write the properties of a symmetric matrix.

(c) Prove that if a determinant has two identical rows (or columns), its value is zero.

(d) Show that

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

(e) Find the rank of the following matrix :

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

3. Answer any *one* of the following questions : 6

(a) The roots of the equation $x^3 - 3x^2 + kx + 3 = 0$ are in AP. Find the value of k and solve the equation.

(b) In $V_3(R)$, where R is the field of real numbers, examine each of the following sets of vectors for linear dependence :

(i) $\{(-1, 2, 1), (3, 0, -1), (-5, 4, 3)\}$

(ii) $\{(2, 3, 5), (4, 9, 25)\}$

(c) Show that the vectors (1, 2, 1), (2, 1, 0), (1, −1, 2) form a basis of R^3.

4. Answer any *two* of the following questions : 3×2=6

(a) Show that if A and B are two idempotent matrices, then $A+B$ will be idempotent, if $AB = BA = 0$.

(b) Show that the transpose of an orthogonal matrix is also orthogonal.

(c) Let A and B be two square matrices of order n. Then prove that

$$\operatorname{tr}(A+B) = \operatorname{tr}(A) + \operatorname{tr}(B)$$

5. Answer any *two* of the following questions : 6×2=12

(a) Define adjoint of a square matrix. If $A_{n \times n}$ is a square matrix of order n, then prove that

$$A(\operatorname{adj} A) = (\operatorname{adj} A)A = |A| I_n$$ 2+4=6

(b) Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$ 6

(c) Solve the following equations by using Cramer's rule : 6

$$x + y + z = 7$$
$$x + 2y + 3z = 16$$
$$x + 3y + 4z = 22$$

6. Answer any *two* of the following questions : $5\times2=10$

(a) Show that the rank of the transpose of a matrix is the same as that of the original matrix.

(b) State and prove the Cayley-Hamilton theorem.

(c) What do you mean by g-inverse? Show that a generalized inverse always exists and is not unique.

(d) Write the quadratic form corresponding to the symmetric matrix :

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 0 & 3\\ 3 & 3 & 1\end{bmatrix}$$

(Old Course)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. Choose the correct answer from the given alternatives in each question : $1\times8=8$

(a) A polynomial of two terms is called a

(i) monomial

(ii) binomial

(iii) trinomial

(iv) None of the above

(b) A polynomial of degree n has how many complex roots?

(i) Exactly n

(ii) $n-1$

(iii) At least $n+1$

(iv) None of the above

(c) If $\begin{bmatrix}5 & k+2\\ k+1 & -2\end{bmatrix}=\begin{bmatrix}k+3 & 4\\ 3 & -k\end{bmatrix}$, then k is

(i) 0

(ii) 2

(iii) -2

(iv) 1

(d) The trace of a square matrix A, denoted by $\operatorname{tr}(A)$, is defined as the

(i) product of diagonal elements

(ii) sum of diagonal elements

(iii) sum of all elements in the matrix

(iv) determinant of the matrix

(e) What happens to the value of a determinant if all its rows are interchanged with its columns?

(i) The value remains unchanged

(ii) The sign of the value changes

(iii) The value is multiplied by -1

(iv) The value becomes zero

(f) The determinant of a skew-symmetric matrix of odd order is always

(i) 0

(ii) 1

(iii) -1

(iv) a perfect square

(g) The rank of a null matrix is always

(i) 0

(ii) 1

(iii) infinity

(iv) undefined

(h) A quadratic form is said to be 'Positive Definite' if all its eigenvalues are

(i) non-negative (≥ 0)

(ii) negative (< 0)

(iii) positive (> 0)

(iv) zero

2. Answer the following questions : $4\times4=16$

(a) Find the values of A, B and C for which

$$A(x-3)(x-1)+B(x+1)(x-1)+C(x+1)(x-3)=6x-10$$

is an identity.

(b) Write the properties of a symmetric matrix.

(c) Show that the matrix

$$A=\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$$

is orthogonal.

(d) Find the rank of the following matrix :

$$A=\begin{bmatrix}-2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1\end{bmatrix}$$

3. Answer any *two* of the following questions : $7\times2=14$

(a) *(i)* Find the roots of the cubic equation $x^3-12x^2+39x-28=0$, given that the roots are in AP. 4

(ii) If the sum of two roots of the cubic equation $x^3 + a_1x^2 + a_2x + a_3 = 0$ is zero, prove that $a_1a_2 = a_3$.

(b) Define vector space. Let R be the field of real numbers. Which of the following are subspaces of $V_3(R)$?

(i) $\{(x, 2y, 3z) : x, y, z \in R\}$

(ii) $\{(x, x, x) : x \in R\}$

(iii) $\{(x, y, z) : x, y, z$ are rational numbers$\}$

(c) In $V_3(R)$, where R is the field of real numbers, examine each of the following vectors for linear dependence :

(i) $\{(-1, 2, 1), (3, 0, -1), (-5, 4, 3)\}$

(ii) $\{(2, 3, 5), (4, 9, 25)\}$

(d) Determine whether or not the following vectors form a basis of R^3 :

$(1, 1, 2), (1, 2, 5), (5, 3, 4)$

4. Answer any *one* of the following questions :

(a) Define an orthogonal matrix. Write the properties of an orthogonal matrix. Show that if A is an orthogonal matrix, then $|A| = \pm 1$.

3+3+3=9

(b) Define singular and non-singular matrices. For what value of x, the following matrix becomes singular? 2+2+5=9

$$\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

(c) Define involutory matrix. Write the properties of involutory matrix. Show that

$$A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

is involutory. 3+3+3=9

5. Answer any *four* of the following questions : 6×4=24

(a) Give the general definition of determinant. Prove that if a determinant has two identical rows (or columns), its value is zero.

(b) Define adjoint of a square matrix. If $A_{n \times n}$ is a square matrix of order n, then prove that

$$A(\text{adj}\, A) = (\text{adj}\, A)A = |A|\, I_n$$

(c) Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

(d) Solve the following equations by using Cramer's rule :

$$x + y + z = 7$$
$$x + 2y + 3z = 16$$
$$x + 3y + 4z = 22$$

(e) What is echelon matrix? Reduce the following matrix into echelon form :

$$A = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$$

(f) Show that the equations :

$$x + 2y - z = 3$$
$$3x - y + 2z = 1$$
$$2x - 2y + 3z = 2$$
$$x - y + z = -1$$

6. Answer any *two* of the following questions : $4\frac{1}{2} \times 2 = 9$

(a) Show that the rank of the transpose of a matrix is the same as that of the original matrix.

(b) State and prove the Cayley-Hamilton theorem.

(c) What do you mean by *g*-inverse? Show that a generalized inverse always exists and is not unique.

(d) Write the quadratic form corresponding to the symmetric matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$

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