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4 SEM TDC GEMT (CBCS) 4.1/4.2/4.3

2026

(May/June)

MATHEMATICS

(Generic Elective)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions.

All symbols have their usual meanings.

Paper : GE–4.1

(Algebra)

1. *(a)* State True or False : 1

   Addition of natural numbers in binary composition is not associative.

   (b) Define centre of a group. 1

   (c) Show that the subset $\{1, -1, i, -i\}$ of the complex numbers is an Abelian group under complex multiplication. 5

   (d) Prove that a non-empty subset H of a group G is a subgroup of G iff

   $$a, b \in H \Rightarrow ab^{-1} \in H$$ 4

(e) Show that the centre of a group G is a subgroup of G. 4

Or

Prove that a subgroup of a cyclic group is cyclic.

2. *(a)* Define a finite group. 1

(b) Describe the symmetries of an isosceles triangle. 3

(c) Prove that every subgroup of an Abelian group is normal. 3

(d) Prove that every quotient group of a cyclic group is cyclic. 5

Or

Show that a subgroup of index 2 in a group G is a normal subgroup of G.

3. Answer any *two* of the following questions : 5×2=10

(a) For any subgroup H of a finite group G, show that $O(G) = O(H)[G : H]$.

(b) Prove that any finite semigroup in which both cancellation laws hold is a group.

(c) Prove that the set of matrices

$$A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

where α is a real number, forms a group under matrix multiplication.

4. *(a)* Let H be a subgroup of G and let a and b belong to G. Then prove that—

(i) $aH = H$ if and only if $a \in H$;

(ii) $aH = bH$ if and only if $a \in bH$. 2+2=4

(b) Find the list of subgroups of $\langle a \rangle$, where a has order 30. 3

(c) Prove that every permutation of a finite set can be written as a product of disjoint cycles. 5

5. *(a)* Prove that every subgroup of an Abelian group is normal. 2

(b) Let G be a group and let H be a normal subgroup of G. Then prove that the set $G/H = \{ah : a \in G\}$ is a group under the operation $(aH)(bH) = abH$. 5

Or

Show that a permutation with odd order must be an even permutation.

6. (a) Define zero divisors. 1

(b) Show that in a ring R, $a(-b) = -(ab)$, for all $a, b \in R$. 2

(c) Prove that the set $R = \{0, 1, 2, 3, 4, 5\}$ is a commutative ring with respect to addition and multiplication modulo 6 as the two ring compositions. 5

(d) Prove that a ring R is without zero divisors if and only if the cancellation laws hold in R. 4

Or

Prove that every field is an integral domain.

7. (a) Define division ring. 1

(b) Show that the intersection of two subrings is a subring. 3

Or

Show that a commutative ring with unity is a field if it has no proper ideals.

(c) Show that the set of numbers of the form $a + b\sqrt{2}$, with a and b as rational numbers, is a field. 5

(d) Find the quotient and remainder upon dividing $f(x) = 3x^4 + x^3 + 2x^2 + 1$ by $g(x) = x^2 + 4x + 2$, where $f(x)$ and $g(x)$ belong to $Z_5[x]$. 3

Paper : GE–4.2

(Application of Algebra)

1. (a) BIBD ৰ সংজ্ঞা লিখা। 2

Define balanced incomplete block design (BIBD).

(b) BIBD ৰ ইন্সিডেন্স মেট্ৰিক্সৰ বিষয়ে লিখা। 2

Explain incidence matrix of a BIBD.

(c) তলৰ যি কোনো দুটা প্ৰশ্নৰ উত্তৰ লিখা : 6×2=12

Answer any *two* of the following questions :

(i) ধৰা হওক, V সংহতিটো vটা মৌলৰে গঠিত, আৰু $D = V^{(k)}$ হৈছে kটা মৌল থকা Vৰ উপ-সংহতিটোৰ সংহতি যাতে $1 < k < v$ হয়। তেন্তে দেখুওৱা যে D এটা BIBD হয় Vৰ কাৰণে যাৰ প্ৰাচল (v, b, r, k, λ), য'ত

$$b = \binom{v}{k}, \quad r = \binom{v-1}{k-1} \text{ আৰু } \lambda = \binom{v-2}{k-2}$$

বিশেষভাৱে দেখুওৱা যে, যদি $k = v-1$, তেন্তে D এটা সমমিতিয় BIBD হয় যাৰ প্ৰাচল

$$(v, v, v-1, v-1, v-2)$$

Let V be a set of v elements, and let $D = V^{(k)}$ be the set of all subsets of V having k elements, $1 < k < v$.

Then show that D is a balanced incomplete block design (BIBD) on V with parameters (v, b, r, k, λ), where

$$b = \binom{v}{k}, \; r = \binom{v-1}{k-1} \text{ and } \lambda = \binom{v-2}{k-2}$$

In particular, if $k = v-1$, then show that D is a symmetric BIBD with parameters $(v, v, v-1, v-1, v-2)$.

(ii) {1, 2, 3, 4, 5, 6} সংহতিটোৰ কাৰণে (6, 10, 5, 3, 2) প্ৰাচলৰ সৈতে এটা BIBD গঠন কৰা।

Construct a BIBD on the set {1, 2, 3, 4, 5, 6} with parameters (6, 10, 5, 3, 2).

(iii) ধৰা হওক $G = \mathbb{Z}$, $S_1 = \{0, 1, 2, 4\}$ আৰু $S_2 = \{0, 3, 4, 7\}$. দেখুওৱা যে S_1 আৰু S_2য়ে পৃথক সংহতিৰ সদস্য হিচাপে দেখুৱায়। ইয়াৰ প্ৰাচল নিৰ্ণয় কৰা।

Let $G = \mathbb{Z}$, $S_1 = \{0, 1, 2, 4\}$ and $S_2 = \{0, 3, 4, 7\}$. Show that S_1 and S_2 form a difference set family, and find its parameter.

2. *(a)* ধৰা হওক C এটা ক'ড যাৰ ন্যূনতম দূৰত্ব হৈছে d. যদি $t = \left[\frac{d-1}{2}\right]$ হয়, তেন্তে দেখুওৱা যে Cয়ে $d-1$ টা ভুলৰ ইংগিত দিব পাৰে, যি কোনো প্ৰেৰিত সাংকেতিক শব্দ (code word)ৰ। 4

Let C be a code with minimum distance d. Let $t = \left[\frac{d-1}{2}\right]$. Then show that C can detect up to $d-1$ errors in any transmitted code word.

(b) তলৰ যি কোনো দুটা প্ৰশ্নৰ উত্তৰ লিখা : 6×2=12

Answer any *two* of the following questions :

(i) প্ৰমাণ কৰা যে Ham (r, q) এটা নিখুঁত ক'ড যাৰ ন্যূনতম দূৰত্ব 3.

Prove that Ham (r, q) is a perfect code with minimum distance 3.

(ii) ক'ড Cৰ ন্যূনতম দূৰত্বৰ সংজ্ঞা দিয়া। $\mathbb{F}_{11}$ ত ক'ড $C\,[10, 8]$ ৰ ন্যূনতম দূৰত্ব নিৰ্ণয় কৰা য'ত parity check matrix হৈছে

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{bmatrix}$$

Define minimum distance of a code C. Find the minimum distance of the [10, 8] code C over $\mathbb{F}_{11}$ with parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{bmatrix}$$

(iii) Cyclic Hamming code Ham (3, 2) ৰ কাৰণে generator মেট্ৰিক্স আৰু parity check মেট্ৰিক্সটো লিখা। সম্পূৰ্ণ ক'ডটোও উলিওৱা।

Write the generator matrix and a parity check matrix for a cyclic Hamming code Ham (3, 2). Obtain also the complete code.

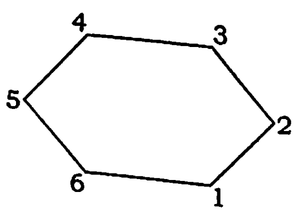
3. *(a)* ধৰা হওক X এটা সংহতি। দেখুওৱা যে Sym (X) হৈছে সমমিতিয় গ্ৰুপ S_X ৰ উপগ্ৰুপ (subgroup). 4

Let X be a set of points. Then show that Sym (X) is a subgroup of the symmetric group S_X.

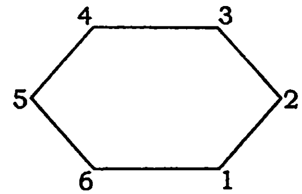
(b) তলৰ যি কোনো দুটা প্ৰশ্নৰ উত্তৰ লিখা : 6×2=12

Answer any *two* of the following questions :

(i) ধৰা হওক, m টা ৰংৰ পৰা এটা নিয়মিত ষড়ভূজৰ শীৰ্ষবিন্দুত ৰং দিয়া হৈছে। সকলো ৰংৰ ভিতৰত ৰংৰ সুকীয়া আৰ্হিৰ সংখ্যা বিচাৰি উলিওৱা :



Suppose each vertex of a regular hexagon is colored by one of m given colors. Find the number of distinct patterns among all colorings :



(ii) পলিয়াৰ উপপাদ্যটো উল্লেখ আৰু প্ৰমাণ কৰা।

State and prove Polya's theorem.

(iii) চাৰিটা বিন্দুৰ অসমৰূপী গ্ৰাফৰ জেনেৰেটিং ফলন $f_4(x)$ টো উলিওৱা।

Find the generating function $f_4(x)$ for the non-isomorphic graphs on four vertices.

4. *(a)* ধৰা হওক A এটা $n \times n$ জটিল মেট্ৰিক্স। দেখুওৱা যে A এটা সমঘাতী মেট্ৰিক্স যদি আৰু কেৱল যদিহে A এটা ডায়েগ'নেল মেট্ৰিক্স diag $(1, \cdots, 1, 0, \cdots, 0)$ ৰ সমান হয়। 4

Let A be an $n \times n$ complex matrix. Show that A is idempotent if and only if A is similar to a diagonal matrix of the form diag $(1, \cdots, 1, 0, \cdots, 0)$.

(b) প্ৰমাণ কৰা যে এটা $n \times n$ মেট্ৰিক্স, Hadamard মেট্ৰিক্স হোৱাৰ প্ৰয়োজনীয় চৰ্তটো হৈছে যে n, 4 ৰ গুণাত্মক, য'ত $n > 2$. 4

Let $n > 2$. A necessary condition for an n-square matrix A to be Hadamard matrix is that n is a multiple of 4. Prove it.

(c) Birkhoff ৰ উপপাদ্যটো উল্লেখ আৰু প্ৰমাণ কৰা। 8

State and prove Birkhoff theorem.

অথবা / Or

প্ৰমাণ কৰা যে প্ৰত্যেক $A \geq 0$ ৰ কাৰণে সদায় এটা অনন্য $B \geq 0$ থাকে যাতে $B^2 = A$ হয়, য'ত A আৰু B দুটা $n \times n$ জটিল মেট্ৰিক্স।

Prove that for every $A \geq 0$, there exists a unique $B \geq 0$ so that $B^2 = A$, where A and B be two $n \times n$ complex matrices.

5. তলৰ যি কোনো দুটা প্ৰশ্নৰ উত্তৰ লিখা : 8×2=16

Answer any *two* of the following questions :

(a) $Ax = b$ ৰ কাৰণে least squares সমাধান উলিওৱা য'ত

Find a least squares solution of $Ax = b$ for

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}$$

(b) আনুমানিক বিপৰীত মেট্ৰিক্স A ৰ মান নিৰ্ণয় কৰা য'ত

Find the approximate inverse of the matrix A for

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 4 \\ 0 & 1 & 1 \end{bmatrix}$$

(c) তলৰ মেট্ৰিক্সটোৰ বাবে শাৰীৰ সৈতে L, D আৰু U উলিওৱা :

Find L, D and U with row for the following matrix :

$$A = \begin{bmatrix} 0 & 0 & 3 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 7 & 0 \end{bmatrix} = LDU$$

Paper : GE-4.3

(Combinatorial Mathematics)

1. *(a)* Find ${}^5C_2$. 1

(b) A task can be done in 6 ways. Another task can be done in 5 ways after the first task has been done. Find the number of ways to do both the tasks. 1

(c) Show that

$${}^nC_r = {}^nC_{n-r}$$ 2

(d) Answer any *two* of the following questions :

(i) State the binomial theorem. Show that for non-negative integer n

$$\sum_{r=0}^{n} 2^r\, {}^nC_r = 3^n$$ 1+3=4

(ii) Find the number of terms in the expansion of $(x+2y+3z)^7$. Also find the coefficient of $x^3y^3z^2$ in the expansion. 2+2=4

(iii) Define factorial polynomial and rising factorial polynomial. Find $S(2, 1)$. 1+1+2=4

2. *(a)* State True or False : 1
If A and B are finite sets with $|A \cap B| = 0$, then $|A \cup B| = |A| + |B|$.

(b) Let A and B be two finite sets such that $|A| = 3$, $|B| = 4$ and $|A \cup B| = 5$. Find $|A \cap B|$. 2

(c) Find d_3, where d_3 is the number of derangements of 1, 2, 3. 3

(d) How many integers between 1 and 200 are divisible by at least one of 3 and 4? 4

3. *(a)* Find the generating function for the sequence (a_n), where $a_n = 1 \ \forall\, n$. 2

(b) Find the sequence whose generating function is

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$ 1

(c) Find the exponential generating function for the sequence (a_n), where

$$a_n = n \ \forall\, n \in \mathbb{N}$$ 2

(d) Find the sequence that corresponds the generating function $G(x) = 3x^3 + e^{2x}$. 3

(e) Find the number of the integral solution of the equation $x+y+z=10$. 4

Or

Find the generating function for the sequence (a_n), where $a_n = \frac{2}{3} + 4n$.

4. *(a)* Define a recurrence relation. 1

(b) Find a sequence such that

$$a_n - 2a_{n-1} = 0, \; a_0 = 3$$ 3

(c) Find the total solution of the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 2^n + 3n$. 6

Or

Using generating function, solve the recurrence relation

$$a_n - a_{n-1} - 6a_{n-2} = 0, \; a_0 = 2, \; a_1 = 1$$

5. *(a)* Define $P(n)$ and $P_k(n)$. Hence find $P(5)$ and $P_3(7)$. 2+2=4

(b) Show that

$$P_3(n) = \left\{\frac{n^2}{12}\right\}$$ 8

Or

Let $A_1, \cdots, A_n$ be finite sets. Show that $A_1, A_2, \cdots, A_n$ have a system of distinct representatives, if

$$\sum_{1 \le i < j \le n} \frac{|A_i \cap B_j|}{|A_i| \cdot |A_j|} < 1$$

6. *(a)* State necklace problem. 2

(b) What is cyclic index of a permutation group? 2

(c) State and prove Burnside's lemma. 1+7=8

Or

Give the proof of Polya's counting method. 8

7. *(a)* Find all the Latin squares of order 3. 4

(b) Define a Hadamard matrix. Give example. 1+1=2

(c) Prove that in a symmetric BIBD, any two blocks have λ treatment in common. 6

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