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**4 SEM TDC MTMH (CBCS) C 9**

**2026**

( May/June )

MATHEMATICS

( Core )

Paper : C–9

**( Riemann Integration and Series of Functions )**

*Full Marks* : 80

*Pass Marks* : 32

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions.*

1. (a) Define refinement of a partition on $[a, b]$. 1

(b) Let $f(x) = x$ be defined on $[0, 1]$. Let the partitions $P = \left\{x_i = \frac{i}{5},\ i = 0, 1, 2, \ldots, 5\right\}$ and $Q = \left\{x_i = \frac{i}{6},\ i = 0, 1, 2, \ldots, 6\right\}$ be defined on $[0, 1]$. Find $L(f, P)$, $U(f, P)$; $L(f, Q)$ and $U(f, Q)$; also find the relation between them. 4

2. (a) Define Riemann sum of $f : [a, b] \to \mathfrak{R}$ on $[a, b]$. Give an example of the Riemann sum if $I = [2, 3]$. 1+1=2

(b) Prove that every constant function is Riemann integrable. 3

**Or**

Let $P = \{[x_i, x_{i+1}], t_i\}_{i=1}^{n}$ be a tagged partition of $[a, b]$. Then show that $S(kf, P) = kS(f, P)$.

(c) Answer any *four* of the following : 5×4=20

(i) Prove that a bounded monotonic increasing function is integrable on $[a, b]$.

(ii) Let $f : [a, b] \to \mathfrak{R}$ be integrable. Then prove that $|f|$ is integrable and that

$$\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|dx$$

(iii) Let $f, g : [a, b] \to \mathfrak{R}$ be integrable and $f(x) \le g(x)$, $\forall x \in [a, b]$. Then show that $\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$.

(iv) Let $f : [a, b] \to \mathfrak{R}$ be integrable. Define $F$ on $[a, b]$ as $F(x) = \int_a^x f(t)\,dt$; $x \in [a, b]$. Show that $F$ is derivable at $c$ and $F'(c) = f(c)$, if $f$ is continuous at $c \in [a, b]$.

(v) Let $f$ be continuous on $[a, b]$. Show that there exists $c \in [a, b]$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$



( Continued )



3. (a) Discuss the convergence of

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$ 3

(b) Answer any *one* of the following : 3

(i) Prove that $\int_a^\infty \frac{c}{x^n}\,dx$ is convergent if $n > 1$ and divergent if $n \le 1$.

(ii) Prove that $B(m, n) = B(n, m)$

(c) Discuss the convergence of beta function. 4

4. (a) Define sequence of function with an example. 1

(b) Show that $\left\{\frac{1}{nx+1}\right\}$ is pointwise convergent but not uniformly convergent. 2

(c) State and prove Cauchy's criterion for uniform convergence. 4

(d) Show that $\sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$ is uniformly convergent on any interval $[a, b]$. 4

(e) Let $\{f_n\}$ be a uniformly Cauchy sequence on $X$, then prove that $\{f_n\}$ is uniformly convergent on $X$. 4



( Turn Over )

(f) Let $f_n : [a, b] \to \mathfrak{R}$ be differentiable. Let there exist functions $f$ and $g$ defined on $(a, b)$ such that $f_n \to f$ and $f_n' \to g$ uniformly on $[a, b]$. Show that $f$ is differentiable and $f' = g$ on $(a, b)$. 5

(g) Let $f_n$ be a sequence of real-valued functions on a set $X$. Assume there exists $M_n \geq 0$ such that $|f_n(x)| \leq M_n$, $\forall n \in N$ and $x \in X$ and that $\Sigma M_n$ is convergent. Prove that the series $\Sigma f_n$ is uniformly convergent on $X$. 5

**5.** (a) Define a power series. 1

(b) Deduce the radius of convergence $R$ of a power series $\Sigma a_n x^n$. 4

(c) State and prove Cauchy-Hadamard theorem. 5

(d) If the series $\sum_{n=0}^{\infty} a_n$ converges, then prove that the power series $\sum_{n=0}^{\infty} a_n x^n$ converges uniformly on [0, 1]. 5

*Or*

Determine the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n (x-a)^n$, where $a_n = \frac{1}{a^n}$.

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